OLD KEYNES AND NEW-FISHER: A MODEL OF ANIMAL SPIRIT DRIVEN RECESSIONS*

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Abstract

This paper analyses a model of recessions caused by non-rational expectations about future output and employment in the economy. Optimal government policy is Old Keynesian — fiscal and monetary expansion. If rational expectations are assumed, optimal government policies reverse in sign, and Neo-Fisherian monetary and fiscal policies are optimal. The relationship between the two models is characterized with a simple diagram, a general equilibrium Keynesian cross, the slope of which is determined by the feedback between current income and expectations of future income. If feedback is low and the slope of the cross is less than 1, normal Keynesian results hold: greater than 1, Neo-Fisherian results are ascendant.

Keywords: Self fulfilling expectations, non-rational expectations, liquidity trap, zero lower bound, Neo-Fisherian policies.

JEL Classification:

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1 Introduction

This paper explores the relationship between two models of recessions caused by low expectations about future output and employment: Old Keynesian models, in which expectations are formed adaptively or extrapolatively, and modern models of Mertens and Ravn (2014) and Benhabib, Schmitt-Grohé and Uribe (2001) where expectations are model consistent ("rational"). Although in both classes of models low expectations cause recessions, the policy conclusions are radically different. While in Old Keynesian models increased government spending and cutting interest rates can boost output, in the modern classes of models the conclusions are exactly the opposite: higher interest rates and lower govenrment spending can raise output and stabilize the economy. This latter properties of modern models are referred to as "Neo-Fisherian" properties.[1]

The paper first builds a model in which the Old Keynesian ideas are analyzed in a modern framework. The model setup is a standard 3 period OLG model as in Eggertsson, Mehrotra and Robbins (2019), which allows for closed form solutions. Agents are permanent income consumers, subject to a borrowing limit; firms are monopolistically competitive. Monetary policy is given by a Taylor rule, and there is a Phillips curve like relation between inflation and unemployment. The main difference with standard models are the agents' expectations about future employment, which are formed either adaptively or based on extrapolative expectations. These non self-fulfilling expectations ensure multipliers have the normal signs, and there are no Neo-Fisherian properties.

If the expectations of this paper's model are instead assumed to be self-fulfilling, two Neo-Fisherian properties emerge: (i) *NF-1*: an interest rate peg eliminates the self-fulfilling equilibrium with high unemployment, and ensures the economy always converges to full employment (ii) *NF-2*: fiscal and monetary expansion cause output to contract. These results are in line with the previous literature on self-fulfilling expectation driven recessions, best exemplified by Schmitt-Grohé and Uribe (2017), Heathcote and Perri (2018), and Mertens and Ravn (2014) .[2]

I characterize why the two Neo-Fisherian properties arise when rational expectations are assumed. I show that *NF-1* is caused by the fact under rational expectations, there is an Euler like relationship between interest rates and the growth rate of output. This ensures that when output is below target, higher interest rates raise output, stabilizing the economy. The *NF-2* property is due to the

[1]The focus of this paper is on the strand of Neo-Fisherian literature concerned with self-fulfilling expectation recessions, as in Schmitt-Grohé and Uribe (2017), Heathcote and Perri (2018), Mertens and Ravn (2014), Bilbiie (2018*b*).

[2]The main difference between these classes of models is that in Schmitt-Grohé and Uribe (2017) the natural rate of interest is not a function of the steady state level of employment, while in Heathcote and Perri (2018) the natural rate of interest is positively related to employment. In Eggertsson, Mehrotra and Robbins (2019) there was also a self-fulfilling equilibrium in which the natural rate of interest was negatively related to employment, but it was not analyzed in detail.

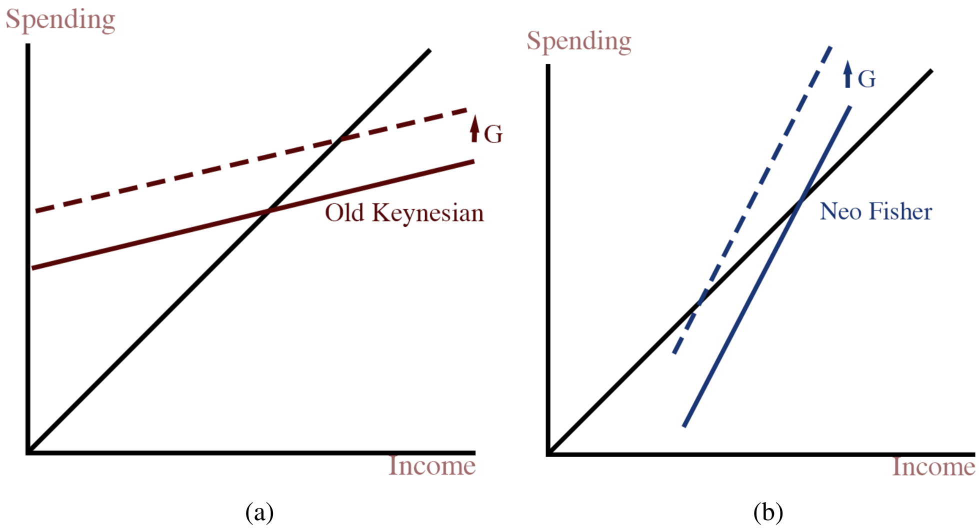


Figure 1: General equilibrium Keynesian cross (a) Old Keynesian properties (b) Neo-Fisherian properties

strong feedback loop between current output and expectations of future output present under rational expectations. The feedback is so strong that there cannot be an equilibrium in which government spending raises output: if it did, it would lead to an ever increasing spiral of higher expectations and higher spending.

The strong feedback loop can be characterized by a simple diagram, a "general equilibrium" Keynesian cross, shown in figure 1. The diagram shows aggregate spending as a function of income. The difference between this diagram and the standard Keynesian cross is that in the general equilibrium version, interest rates and expectations of output are a function of current income. Under rational expectations of the Mertens & Ravn variety, the slope of the GE expenditure function is greater than 1, thus it intersects the 45 degree line from below (panel b). This ensures that anything that in partial equilibrium would boost spending in general equilibrium will lead to a decrease in income and spending. This characterization shows that NF states only occur in a sense when the economy is "too Keynesian", and spending reacts too much to changes in income.

In the baseline expectations of the Old Keynesian formulation, expectations of future output are formed based on past output only, which ensures that the slope of GE Keynesian cross is less than 1 and the Neo-Fisherian properties go away. Under adaptive or extrapolative expectations, the feedback loop between current output and expectations of output is broken, eliminating *NF-2*. In addition, without rational expectations the growth rate of output is no longer dominated by the Euler equation, and there is a breakdown in the relationship between real interest rates and the growth rate of output. In fact, high real interest rate often lead to a decrease in the growth rate of output. Higher real interest rates depress contemporaneous output, which leads to lower expectations of future output, which in turn also depresses output growth. Without the positive relationship between real interest rates and output growth, *NF-1* no longer holds. Finally, under these non-rational expectations the economy can no longer reach the low-employment steady state of Schmitt-Grohé and Uribe (2017) and Heathcote and Perri (2018), as the point is completely unstable.

The new understanding of NF properties allows this paper to specify, for the first time, a class of non-rational expectations under which the NF properties hold. In particular, if expectations of future output are based on current output, this reintroduces a feedback loop between current output and expectations. If the feedback loop is strong enough, the slope of the GE Keynesian cross will be greater than one, and *NF-2* properties re-emerge. In addition, *NF-1* properties also resurface; because *NF-2* holds, higher interest rates raise output and expectations of output, leading to actual output growth. The higher interest rates of a peg will again raise output when it is low, stabilizing the economy. For similar reasons, with the additional feedback loop, the "low-employment" steady state of Schmitt-Grohé and Uribe (2017) becomes stable again.

This paper identifies several strange properties of models with NF states. First, NF states require improbably large movement of expectations to changes in government spending or interest rates. Second, NF properties are only present after a type of singularity point in which the government spending multiplier jumps from ∞ to $-\infty$. Up to this singularity point, larger response of expectations to current output will increase multipliers. At the singularity, however, expectations grow too much to be consistent with any equilibrium with normal signed multiplier, and the Neo-Fisherian equilibrium is the result. There is a third perverse property: any positive shock to expectations under Neo-Fisherian conditions ultimately leads to lower expectations in equilibrium — the shock to expectations leads to a decrease in output, which feeds into lower future expected output.

The properties of the OK model are of interest of itself, and the recession are analyzed and contrasted with recessions from the New Keynesian or disequilibrium literature. The fundamental cause of recessions is a lack of effective demand caused by low expectations, not sticky prices or the zero lower bound. In the model, even if "the prices are right" — real wages and real interest rates at their natural rate — the economy can be in recession. Under flexible wages the results of the model are unchanged. There can also still be recessions if the zero lower bound constraint is relaxed. The zero lower bound does make recessions worse, because lower real interest rates boost output in this model.[3]

The paper is closely related to the adaptive learning literature, which has studied extensively how adaptive learning affects New Keynesian and Neo-Fisherian models, and conclusively shown that in most cases Neo-Fisherian results do not hold under adaptive learning (Benhabib, Evans and Honkapohja (2014), Christiano, Eichenbaum and Johannsen (2018), Evans and McGough (2018)). This paper generalizes these results to show that Neo-Fisherian results are not caused

[3]That said, in the context of the model, with real rates low enough recessions can be prevented.

by rational expectations in and of themself, but by the high degree of feedback between spending and income that are present under rational expectations. In doing so, I show for the first time a form of non-rational expectations that generates Neo-Fisherian properties. This broader framework allows a unified treatment of the two strands of Neo-Fisherian results (*NF-1* and *NF-2*) and the conditions under which they do or do not hold.

The paper is also closely related to Evans, Honkapohja and Mitra (2016), which also studies the effects of pessimistic expectation shocks under non-rational expectations. A key difference is this paper's focus on characterising why the standard multipliers that hold under non-rational expectations don't hold under rational expectations. A second difference is the OLG setup of this paper, which greatly simplifies the dimentionality of expectations and dynamics of the model.

In a series of articles, Farmer and Plotnikov (2010) and Farmer (2013) models depressions caused by self fulfilling beliefs about the stock market and economy. A key difference between Farmer and this paper is that in Farmer there is no monetary authority that cuts interest rates if output is below potential. Having this property restricts the number of long run equilibria in this paper. In Farmer there is nothing preventing the economy settling at 5% unemployment, 15% unemployment, or any level. In this paper, the economy could never settle there because the central bank would cut interest rates. Another difference is that in Farmer, output is 100%, or close to 100% determined by beliefs. Government spending cannot raise employment in this model, it can only lead to crowding out. In my model beliefs are a partial determinant of output, but there are other determinants as well that can raise output.

A sub-theme in recent macroeconomic literature has been the strong feedback mechanisms in New-Keynesian models, which causes surprising results such as the "forward guidance puzzle" and Neo-Fisherian results (see, for example, McKay, Nakamura and Steinsson (2016), Gabaix (2016), and Bilbiie (2018*b*)). These papers have found that dampening the feedback mechanisms can help to get rid of some of the unrealistic properties. A unique aspect of this paper is the characterization of the feedback process in terms of a General Equilibrium Keynesian Cross. This gives a very intuitive way of understanding why when feedback gets too high, the model switches to a Neo-Fisherian state.

2 Model with non-rational expectations

Individuals live for three periods: young (y), middle aged (m), and old (o). Agents choose consumption to maximize their lifetime utility. Young individuals are constrained by a debt limit, which is a function of the total output of the economy and the interest rate: $C_t^y \leq D * Y_t/\widehat{R_t}$, where D is the debt limit parameter, $\widehat{R_t}$ the subjective expected real interest rate. To simplify the dynamics, we assume that D is low enough that the constraint is always binding: $C_t^y = D * Y_t/\widehat{R_t}$. In the second and third period of life individuals are unconstrained, and maximize subjective utility

$$U_t = \log C_t^m + \beta \log \widehat{C_{t+1}^o}.$$

The only uncertainty for agents is for the level of output next period, Y_{t+1}, and the gross inflation rate next period π_{t+1}, which in turn affects the expected real interest rate $\widehat{R_t}$. Agents form expectations $\widehat{Y_{t+1}}$ and $\widehat{\pi_{t+1}}$ about output and prices. We assume that agents act as if $\widehat{Y_{t+1}}$ and $\widehat{\pi_{t+1}}$ will occur with certainty. This assumption is made for simplicity, since the focus of the model is on whether agents have high or low expectations, and not the variance of expectations.

Young agents do not work. The middle aged generation inelastically supply γ units of labor, and old individuals supply $(1-\gamma)$ units of labor. Under full employment, total labor supplied is equal to 1. If there is unemployment, with labor demand $L_t < 1$, individuals are off their labor supply curves, and we assume that the relative fraction of labor supplied by the different generations is unchanged. The middle generation supplies γL_t units of labor, and the old generation $(1-\gamma)L_t$ units.

There are profits Π_t in the economy, and are distributed to the middle and old generation in proportion to their labor earnings: $\Pi_t^m = \gamma \Pi_t$, $\Pi_t^o = (1-\gamma)\Pi_t$. As will be seen, labor is the only factor of production, and thus total income and output in the economy is given by $Y_t = w_t L_t + \Pi_t$. The budget constraint in middle and old age can then be written as

$$C_t^m + B_{t+1}^o / \widehat{R_t} = \gamma w_t L_t + \gamma \Pi_t + B_t^m = \gamma Y_t + B_t^m \quad (1)$$

$$C_t^o = (1-\gamma) w_t L_t + (1-\gamma)\Pi_t + B_t^o = (1-\gamma) Y_t + B_t^o, \quad (2)$$

where B_{t+1} is the face value of real bonds purchased at time t, at a price $1/\widehat{R_t}$. $\widehat{R_t}$ is the agents' subjective expectation of the gross real interest rate, which we assume equals the gross nominal interest rate times expected inflation: $\widehat{R_t} = I_t \widehat{\pi_{t+1}}$.

Given the constrained maximization problem, optimal consumption is

$$\begin{aligned} C_t^y &= D * Y_t / \widehat{R_t} \\ C_t^m &= \frac{1}{1+\beta}(\gamma Y_t + B_t^m + (1-\gamma)\widehat{Y_{t+1}}/\widehat{R_t}) \\ C_t^o &= B_t^o + (1-\gamma) Y_t \end{aligned}$$

2.1 Subjective Expectations

Agents form expectations of future output adaptively, with expectations revised in accordance with the expectation error and a shock ϵ_t^e:

$$\widehat{Y_{t+1}} = \lambda Y_{t-1} + (1-\lambda)\widehat{Y}_t + \epsilon_t^e \quad (3)$$

We assume that in forming expectations of the future, agents do not observe current output Y_t, and thus expectations of the future are based on output in the past, and not on the present. In particular, in period t, expected output in period $t+1$ is a function of output last period Y_{t-1} and last period's level of expectations for output in the present, $\widehat{Y_t}$.

This assumption could be motivated through delays in economic data and measurement. However, we primarily use this formula for analytical and theoretical clarity: it eliminates the feedback loop between current output and subjective expectations of future output. Later in the paper we will analyze in detail the cases in which expectations are based on current output:

$$\widehat{Y_{t+1}} = \lambda Y_t + (1-\lambda)\widehat{Y_t} + \epsilon_t^e \tag{4}$$

This formulation reintroduces the feedback loop between current output and expectations of future output, and leads to very different results.

For our analytic results, we assume that agents form expectations of inflation myopically:

$$\widehat{\pi_{t+1}} = \pi_t \tag{5}$$

This is made for simplicity, since with myopic expectations there is not a state variable for $\widehat{\pi_t}$. In simulations, the results are very similar if expectations of inflation are formed adaptively or through extrapolative expectations.

2.2 Monetary Policy

Following Eggertsson, Mehrotra and Robbins (2019), monetary policy is set with a Taylor rule, where the nominal interest rate is set as a function of the ratio to the inflation rate to the target inflation rate:

$$1 + i_t = max(1, (1+i^*)(\frac{\pi_t}{\pi^*})^{\phi_\pi}) + \epsilon_t^i$$

Here ϵ_t^i is a monetary policy shock. We assume the Taylor principle is satisfied, and $\phi_\pi > 1$. We set $(1+i^*) = R^* \cdot \pi^*$, the full employment nominal interest rate given the inflation target.

Given this policy rule, when inflation is below target, initially the real interest rate declines when inflation declines, as the central bank lowers the nominal interest rate more than one for one for inflation. At the ZLB, however, the nominal interest rate cannot move, and thus lower inflation leads to a higher real interest rate.

2.3 IS-Exp curve

The aggregate expenditure function in the economy is given by the sum of spending by all generations:

$$Spend_t(Y_t) = \nu Y_{t-1} + \overline{MPC}(\widehat{R_t})Y_t + \chi(\widehat{R_t})\widehat{Y_{t+1}} \tag{6}$$

Here ν[4] represents the propensity to consume out of past income (which affects spending through the accumulation of assets), $\overline{MPC}(\widehat{R_t})$ the propensity to consume out of current income, and $\chi(\widehat{R_t})$[5] the propensity to consume out of expected future income. $\overline{MPC}(\widehat{R_t})$ is a weighted average of the individual generations' marginal propensities to consume out of current income. The weights are relative shares of income of the different generations.[6]

In equilibrium aggregate spending equals aggregate income. Solving for Y_t yields the Investment-Savings-Expectations (IS-Exp) curve.[7]

$$Y_t = \alpha(\widehat{R_t}) \left(\nu Y_{t-1} + \chi(\widehat{R_t}) \widehat{Y_{t+1}} \right) \tag{7}$$

The partial equilibrium Keynesian multiplier, $\alpha(\widehat{R_t})$,[8] equals $\frac{1}{1-\overline{MPC}(\widehat{R_t})}$.

Figure 2b, blue line, shows the short run IS-Exp curve in inflation-output space.[9] The short run IS curve shows a negative relationship between real interest rates and output. With a Phillips curve, real rates will increase when inflation is above target, decreasing employment and output — thus above the kink, the blue line slopes downward. When inflation is below target the real rate will initially decline until the ZLB is hit. After the ZLB binds, lower inflation will lead to higher real rates, lowering employment — thus the blue line slopes upwards below the kink.

2.4 Aggregate supply

2.4.1 Market structure

There is a unit mass of monopolistically competitive final goods firms that produce differentiated goods with labor. The final good composite is the CES aggregate of these differentiated final goods, which are indexed by i:

$$Y_t = \left[\int_0^1 y_t^f(i)^{\frac{\Lambda_t - 1}{\Lambda_t}} di \right]^{\frac{\Lambda_t}{\Lambda_t - 1}}.$$

Final goods firms set prices in each period, and face a demand curve of the form $y_t^f(i) = Y_t^D \left(\frac{p_t(i)}{P_t} \right)^{-\Lambda}$, where Y_t^D is aggregate demand for the composite good

[4] $\nu = \frac{\beta D}{1+\beta}$.

[5] $\chi(\widehat{R_t}) = mpc^m(1-\gamma)/\widehat{R_t}$

[6] $\overline{MPC}(\widehat{R_t}) = mpc^y \frac{D}{\widehat{R_t}} + mpc^m \gamma + mpc^o(1-\gamma)$.The weights sum to more than one because the young generation borrows. The marginal propensities to consume of the young, middle aged, and old are 1, $\frac{1}{1+\beta}$, and 1, respectively.

[7] This equation uses the fact that in equilibrium, total borrowing of the young generation must equal lending from the middle ($B_t^m = -DY_t = -B_t^o$).

[8] Exact formula $\alpha(\widehat{R_t}) = \frac{1}{\left[1 - mpc^y \frac{D}{\widehat{R_t}} - mpc^m \gamma - mpc^o(1-\gamma)\right]}$.

[9] We use the following calibration for all empirical exercises: $\beta = .29$, $\gamma = .9$, $\lambda = .4$, $D = .1$, $\pi^* = 1.02$, $\phi_\pi = 1.5$, $\kappa = .08$.

from consumers, P_t is the nominal price index of the final good aggregate and Λ is a measure of a firm's market power.[10]

Each final goods producer uses labor to produce output, according to a linear technology function $y_t(i) = L(i)$. A final goods firm chooses real prices $\frac{p_t(i)}{P_t}$ and $y_t^f(i)$ to maximize real profits, subject to the production constraint. The marginal cost of producing a unit of final good is the real wage w_t. They thus maximize

$$\Pi_t(i) = \frac{p_t(i)}{P_t} Y_t^D \left(\frac{p_t(i)}{P_t}\right)^{-\Lambda} - w_t Y_t^D \left(\frac{p_t(i)}{P_t}\right)^{-\Lambda} \tag{8}$$

The optimality condition for the real price of the firm's good is a markup $\mu \equiv \frac{\Lambda}{\Lambda-1}$ over marginal cost (which is the real wage): $\frac{p_t(i)}{P_t} = \mu w_t$. Since the price labor is the same, all final goods firms make the same pricing decisions, and thus $p_t(i) = P_t$, yielding $w_t = \frac{1}{\mu}$.

2.4.2 Inflation

We assume a Phillips curve relationship between inflation and unemployment:

$$\pi_t - \pi^* = a_1(Y_t - Y^*) + a_2(\widehat{\pi_{t+1}} - \pi^*).$$

Under our simplified assumed inflation expectations, $\widehat{\pi_{t+1}} = \pi_t$, we have

$$\begin{aligned}(\pi_t - \pi^*)(1 - a_2) &= a_1(Y_t - Y^*)\\ (\pi_t - \pi^*) &= \kappa(Y_t - Y^*) = \kappa(Y_t - 1)\end{aligned}$$

where $\kappa \equiv a_1/(1 - a_2)$. Here κ determines how strongly inflation decreases when there is unemployment. Figure 2a, red line, shows the upward sloping aggregate supply curve in inflation-output space.

The non-forward looking Phillips curve is equivalent to the relationship between inflation and unemployment in Eggertsson, Mehrotra, and Robbins (2019), Schmitt-Grohé and Uribe (2017), Bilbiie, Monacelli and Perotti (2019), and Bilbiie (2018*b*), and microfounded in Bilbiie (2018*a*). The simplifying assumption of no forward looking term in the Phillis curve cuts back on the feedback between output and expectations of output in the model, and is not crucial for the results.

[10] The price index for the final aggregate is given by $P_t = \left(\int_0^1 p_t(i)^{1-\Lambda_t} di\right)^{\frac{1}{1-\Lambda_t}}$.

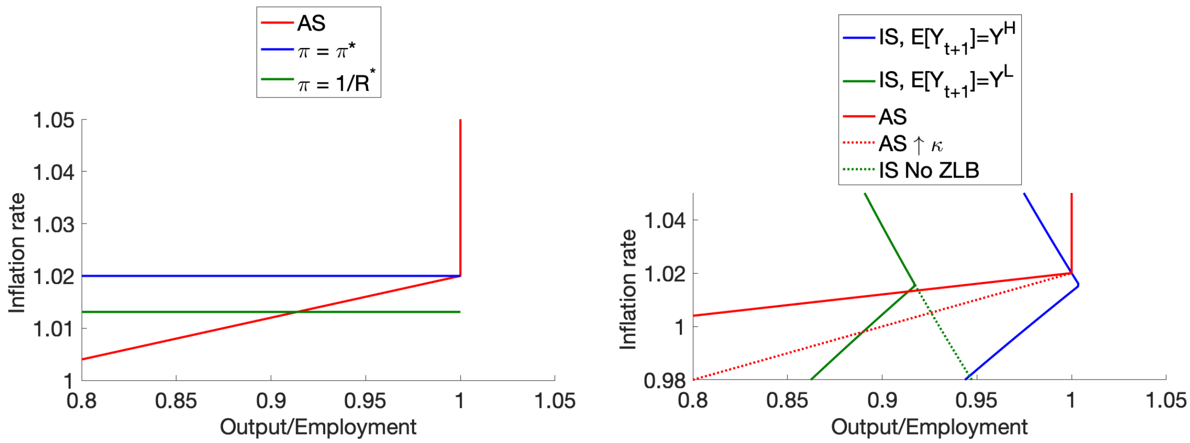


Figure 2: (a) long run aggregate supply and IS curve (b) short run IS curves under different expectations of Y_{t+1}.

2.5 Temporary Equilibrium

The economy reduces to a six equation system:

$$Y_t = \alpha(\widehat{R_t})\left(\nu Y_{t-1} + \chi(\widehat{R_t})\widehat{Y_{t+1}}\right) \tag{9}$$

$$1 + i_t = max(1, (1 + i^*)(\frac{\pi_t}{\pi^*})^{\phi_\pi}) + \epsilon_t^i \tag{10}$$

$$\widehat{R_t} = (1 + i_t)/\widehat{\pi_{t+1}} \tag{11}$$

$$\pi_t = \pi^* - \kappa(1 - Y_t) \tag{12}$$

$$\widehat{\pi_{t+1}} = \pi_t \tag{13}$$

$$\widehat{Y_{t+1}} = \widehat{Y}_t + \lambda(Y_{t-1} - \widehat{Y}_t) + \epsilon_t^e. \tag{14}$$

The first three equations are combined to form the IS-Exp curve in $\pi - Y$ space, the fourth equation is the AS curve, and the fifth and sixth determine the dynamics of expectation formation. A "temporary" equilibrium, in the Hicks (1939) sense, for the economy occurs at the intersection of the AS and IS-Exp curve. At this point, given the current state of expectations, inflation and output are set at the level which set aggregate supply equal to aggregate demand. Outside the steady state, this is depicted in figure 2b, solid blue line. From any initial Y_{t-1} and $\widehat{Y_{t+1}}$, there is a unique intersection of the IS-Exp curve and AS curves.

In a steady state, output and expectations of output are constant and identical. Setting these equal in equation 7 yields the unique equilibrium real interest rate $R^* = \frac{D(1+\beta)+(1-\gamma)}{\beta(\gamma-D)}$.

In general, there are an infinite number of combinations of nominal interest rates and inflation rates that will achieve this real interest rate. However, given the Taylor rule of the monetary authority, there are only two ways for the nominal interest rate to be constant in the steady state. Either inflation is on target at $\pi^* \equiv \pi^H$, or the ZLB is binding, $i = 0$. If the ZLB is binding, then the inflation rate consistent with the natural rate of interest R^* is equal to the inverse of the natural rate, $1/R^* \equiv \pi^L$.

Figure 2a shows the two long-run IS curves in inflation-output space, one at inflation rate π^H, one at π^L. Two steady state equilibria are the intersections of these long run IS-Exp curves with the AS curve. These two equilibria are equivalent to the two equilibria in Schmitt-Grohé and Uribe (2017). We refer to output and inflation in the full employment steady state as Y^H and π^H, and in the low steady state Y^L and π^L.

There is also a third steady state, the "zero" steady state, where output drops to zero: we denote this state by Y^0. As will be seen, if expectations drop low enough output in the economy can drop to this zero steady state.

3 Expectation driven recessions

Negative shocks to expectations of future employment and output can drive the economy to recession. This result will hold as long as there is not too much feedback between income and spending in the economy, i.e. the slope of the GE Keynesian expenditure function less than 1.

Proposition 1. *If*

- *Expectations are formed based on past output* Y_{t-1}
- *The slope of the GE expenditure function is less than 1:* $\frac{\partial Spend_t}{\partial Y_t} < 1$

then $\frac{\partial Y_t}{\partial \epsilon_t^e} \geq 0$*, and a negative shock to expectations leads to a decrease in output.*

Proof. See appendix. □

Negative beliefs about future spending lead to a cut in spending, which cycles back into lower income and a further cut in spending through the standard multiplier process. The condition on the slope of the GE Keynesian expenditure function ensures that there is not too much feedback between income and spending.

Figure 2b illustrates proposition 1. The blue line is the IS-Exp under high expectations, however a negative shock to expectations shifts the curve to the left, lowering output and inflation, as seen by the green line.

The slope of the GE Keynesian expenditure is given by

$$\begin{aligned}\frac{\partial\ Spend_t}{\partial Y_t} &= \overline{MPC}(\widehat{R_t}(Y_t)) + \frac{\partial \overline{MPC}}{\partial \widehat{R_t}} \frac{\partial \widehat{R_t}}{\partial Y_t} Y_t \\ &\quad - \widehat{R_t}^{-1} \chi(\widehat{R_t}) \widehat{Y_{t+1}} \frac{\partial \widehat{R_t}}{\partial Y_t} \overset{?}{<} 1 \qquad (15)\end{aligned}$$

When income increases, spending changes from the partial equilibrium effect (the first term), as well as two general equilibrium effects that flow through changes in interest rates. First, increases in the interest rate lower the amount the younger generation can borrow, lowering the $\overline{MPC}$. Second, higher interest rates lower lower consumption of the middle agent through substitution effects.

Under adaptive expectations based on past output, the second two terms are small and often negative in magnitude, but without further assumptions inequality 15 does not always hold and there can be Neo-Fisherian results. If we fix the state of the economy and expectations, we can provide conditions on parameters that ensure 15 holds.[11]

3.1 Dynamics

If a negative shock to expectations is small, the system will return to full employment.

Proposition 2. *If*

- *Expectations are formed based on past output* Y_{t-1}
- *The slope of the GE expenditure function is less than 1:* $\frac{\partial Spend_t}{\partial Y_t} < 1$

then there is a unique solution path to Y^H *in the vicinity of* Y^H.

Proof. See appendix. □

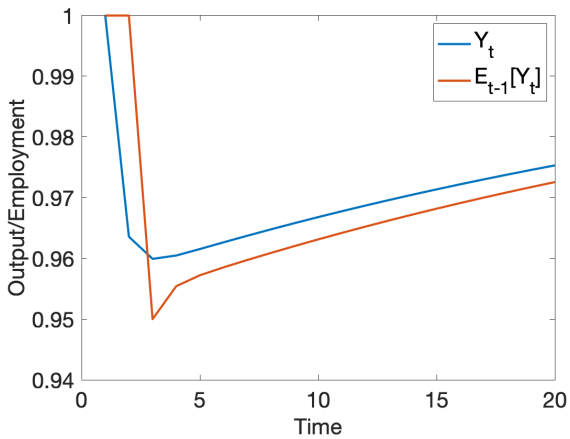
Under the formulated expectations, there are two state variables, Y_{t-1} and $\widehat{Y_t}$ and no jump variables, thus absent any shocks dynamics of the model depend purely on initial conditions. If the system is shocked away from full employment, the lower real interest rate from the Taylor rule will boost output and eventually return the system to Y^H.

Figure 3a shows the effect of a small shock to expectations. In period 1 the economy is at full employment, but in period two there is a small negative expectation shock, $\epsilon_2^e = -.05$, meaning expected output in period 3 is $\widehat{Y_3} = .95$. The lower expectations lead to a drop in output in period 2 and subsequent periods. The lower expectations are somewhat, but not completely self fulfilling —- actual output $Y_3 \approx .96 > \widehat{Y_3}$.

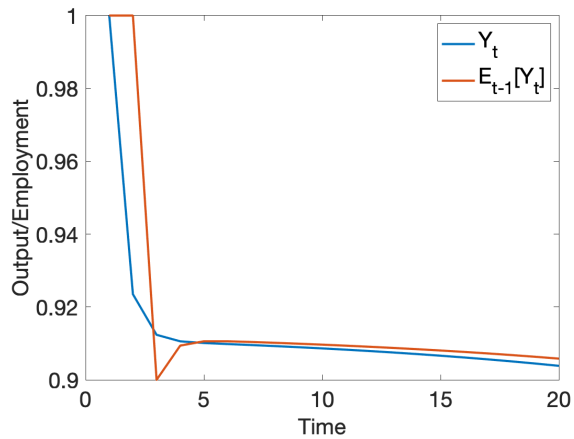
Although the system eventually returns to full employment, this may take a considerable amount of time. Lower output in the previous period and lower expectations are significant drags on output. The only thing pulling up the system are lower interest rates, but depending on the elasticity of intertemporal substitution, lower rates may only have small effects.

If a shock to expectations is large enough, however, the system will not return to full employment, but will slowly tend towards zero. In particular, if $Y <$

[11] If $Y_t = Y^H$, inequality 15 holds under all parameter values.In this case, the second and third terms of 15 are negative. More generally, if the ZLB does not bind the second and third terms will be negative. If $Y_t < Y^H$, $\widehat{Y_{t+1}} < Y^H$, inequality 15 will hold if $\kappa < \frac{\beta}{1+\beta}$ and $\kappa < \frac{1}{R^*}$.



(a)



(b)

Figure 3: Transition dynamics, adaptive expectations, $\lambda = .5$. (a) Small shock, output returns to Y^H(b) Large shock, output goes to Y^0.

Y^L, then inflation is low enough such that $R > R^*$. With higher real interest rates, output is depressed even further, which further depresses expectations, eventually leading output and inflation lower and lower.

Figure 3b shows the transition dynamics of the system for a large negative shock to expectations. In this case the shock is large enough so that $Y_2 < Y^L$. In period 3, the higher real interest rate depresses output even further, and the system slowly starts to converge towards zero output. Once again, the expectation shock is almost but not quite self-fulfilling, with $Y_3 > \widehat{Y}_3$. After period 4, however, expectations of output are generally above output.

A "death spiral" towards zero output seems on the face of it to be an unrealistic property of the model. It should be noted that it generally takes many periods for the economy to converge to zero, and it assumes no other shocks to expectations or government actions. It also assumes that deflation continues, and that expectations continue to degrade. If there is a minimum level of expectations $\bar{\widehat{Y}}$ such that $\widehat{Y}_t > \bar{\widehat{Y}}$, output would instead converge to some positive value slightly lower than $\bar{\widehat{Y}}$. This is the assumption of Evans, Honkapohja and Mitra (2016).

3.2 Cause of recessions

The fundamental cause of recessions in the model is a lack of effective demand, and not sticky prices or the ZLB.

Under flexible nominal or real prices the model still generates long lasting recessions. If nominal prices are more flexible, this will increase the steepness of the AS curve. As seen in figure 2b, red dashed line, this will only decrease employment and output in the recession.

The real wage in our model is flexible and set equal to the marginal product of labor divided by the markup, which is a constant $\frac{1}{\mu}$. However, even if real wages were allowed to fall as unemployment increases, i.e. μ was made a function of Y_t, this would not affect the equilibrium of the model. The AD curve is only a function of aggregate output, and is not a function of distribution between the owners and capital and workers. A lower real wage decreases aggregate demand, but this is exactly cancelled out by the higher profits. As specified, aggregate supply is thus subservient to aggregate demand. Even at the "right prices", i.e. R^*, w^*, if expectations are low enough there will be a recession.

Proposition 1 holds even with no zero lower bound constraint. This can be seen by eliminating the kink in the IS-Exp curve, seen in figure 2b, green dashed line: a negative shock to expectations will still shift the IS-exp curve to the left leading to lower employment. We do note, however, that the ZLB does exacerbate recessions. The elimination of the ZLB constraint has one additional substantive implication: it eliminated the stability of the Y^0 steady state, and ensures Y^H is the unique steady state.

4 Fiscal and monetary policy

The government spends G_t, and raises revenue through taxes T_t and issuing government bonds B_t^G. The government budget constraint is given by:

$$B_{t+1}^G/\widehat{R_t} = B_t^G + G_t - T_t$$

We focus on a fiscal policy in which a fraction of the total taxes is paid by each generation to balance the the budget: τ_t^y, τ_t^m, and τ_t^o. For simplicity, we assume the government does not tax the old: $\tau_t^o = 0$.[12]

The IS-Exp Curve then becomes a function of government policy:

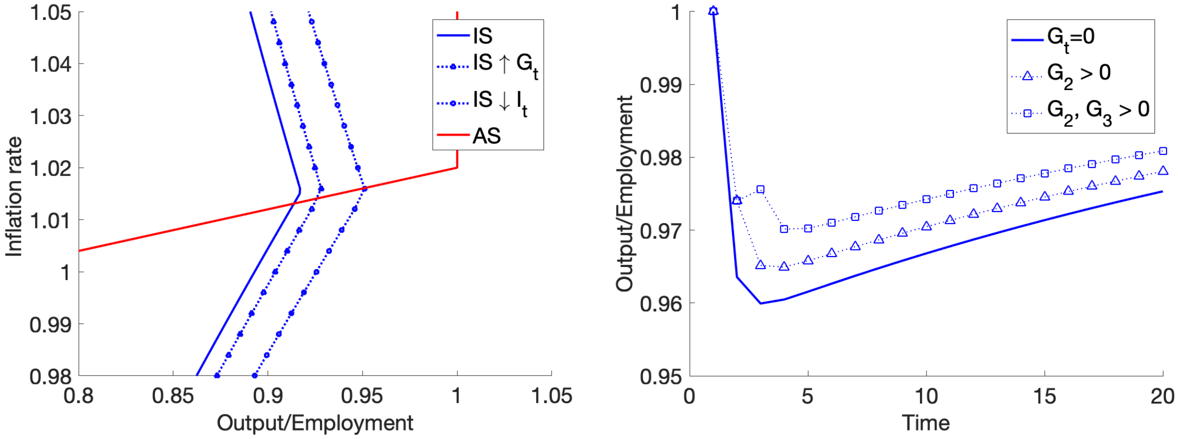
$$Y_t = \alpha(\widehat{R_t}) \Big(\nu Y_{t-1} + \chi(\widehat{R_t}) \widehat{Y_{t+1}} +$$
$$\Theta(\tau_t) G_t - \Upsilon(\tau_t) B_t^G + \Upsilon(\tau_t) \frac{B_{t+1}^G}{\widehat{R_t}} \Big) \tag{16}$$

Here $\Upsilon(\tau_t) = \tau_t^y + \frac{1}{1+\beta}\tau_t^m$, $\Theta(\tau_t) = 1 - \tau_t^y - \frac{1}{1+\beta}\tau_t^m$.

Figure 4a shows the aggregate supply and IS-Exp curve for the model with government spending. An increase in government spending shifts the IS curve to the right, boosting output. The overall government multiplier depends on (i) the normal multiplier $\alpha(\widehat{R_t})$ (ii) $\Theta(\tau_t)$, the distribution of taxes across generations (iii) B_{t+1}^G and B_t^G, the amount of spending which is financed by borrowing. As long as some tax falls on middle generation, $\Theta(\tau_t) > 0$ and the overall multiplier will be positive.

Proposition 3. *If*

[12] This assumption simplifies the dynamics. It is not a completely innocuous assumption: it cuts down the amount of Ricardian equivalence in the model, since for the middle generation, an increase in future taxes no longer depresses spending.



(a) (b)

Figure 4: (a) IS curves, with and without tax financed government spending (b) Transition path, adaptive expectations, $\lambda = .4$.

- *Expectations are formed based on past output* Y_{t-1}
- *The slope of the GE expenditure function is less than 1:* $\frac{\partial Spend_t}{\partial Y_t} < 1$
- *The increase in government spending is tax financed, i.e. there is no change in* B^G_{t+1}

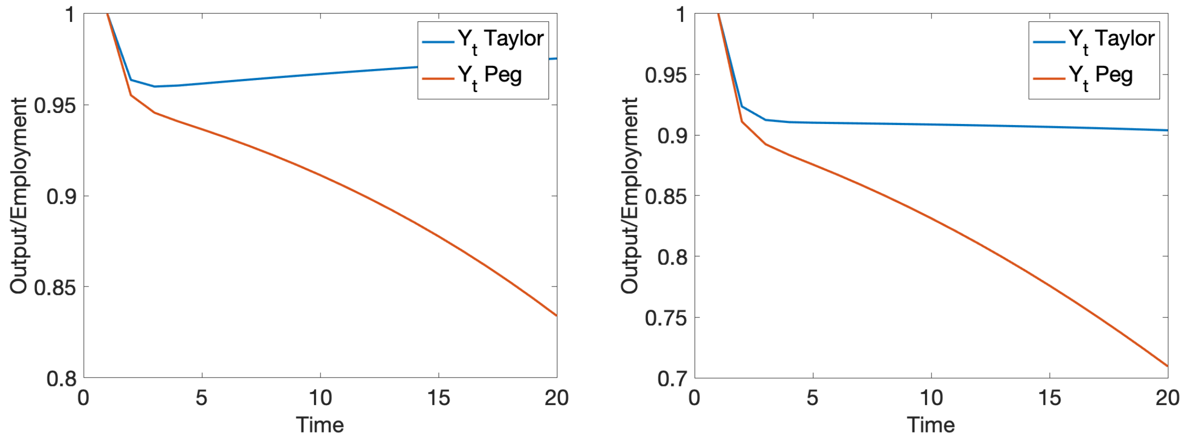
then $\frac{\partial Y_t}{\partial G_t} > 0$*, and government spending increases output.*

Proof. See appendix. □

Note that government spending will lift spending both on impact and *along the entire transition path*. This occurs because higher income in the past means higher level of output and expectations in the future, and thus the higher output on impact Y_t will translate into higher output for all future periods. Figure 4b shows a particular transition path. The initial point of the economy is at full employment at time 1, and then the economy is hit by a negative expectations shock at time 2. The solid blue line shows the evolution of output without government spending. A one time increase in government spending in period 2, shown by the triangle markers, shifts output upwards along the path. The same size increase in both period 2 and period 3, shown with the square markers, shifts output even further.

4.1 Monetary policy shock

A negative shock to nominal interest rates ϵ^i_t shifts the IS-Exp curve to the right, leading to an increase in output. This is seen in figure 4a, triangle markers. A negative shock to interest rates thus has the same sign effect as an increase in government spending.



(a) (b)

Figure 5: Dynamical system, adaptive expectations, Taylor rule and interest rate peg $I_t = I^*$. (a) Small shock $e_2^e = -.05$ (b) Large shock $e_2^e = -.1$.

Proposition 4. *Under adaptive or extrapolative expectations based on past output, if* $\frac{\partial Spend_t}{\partial Y_t} < 1$ *then* $\frac{\partial Y_t}{\partial \epsilon_t^i} < 0$.

Proof. See appendix. □

4.2 Interest rate peg

We now consider monetary policy in which nominal interest rates are pegged, $I_t = R^* \cdot \pi^*$. An interest rate peg always raises real interest rates relative to a Taylor rule. Given our assumed expectations, moving to an interest rate peg will tend to shift the IS-Exp curve to the left, leading to a decline in output.

Figure 5a compares simulations for a Taylor rule versus an interest rate peg, under the same small shock as in section 3.1: $\epsilon_2^e = -.05$. Under a Taylor rule, the system converges to the Y^H. Under an interest rate peg, output instead converges to Y^0.

We can actually say more about output along the path under an interest rate peg. The peg destabilizes the Y^H steady state, since if output is just below full employment higher real interest rates will cause output to decline. The only steady state in the economy under a peg is Y^0, since once output falls below full employment interest rates cannot adjust to raise output.

Proposition 5. *Under backwards looking adaptive or extrapolative expectations and an interest rate peg* $I_t = I^*$, *if* $\frac{\partial Spend_t}{\partial Y_t} < 1$ *then* Y^H *is unstable, and* $Y = 0$ *is the only stable steady state. Thus for any initial conditions, the economy converges to* $Y = 0$.

Proof. See appendix. □

5 Model under rational expectations

Under rational expectations, the model's dynamics closely resembles that of Schmitt-Grohé and Uribe (2017), Mertens and Ravn (2014), and Heathcote and Perri (2018), inclusive of Neo-Fisherian results of monetary and government policy.

The first differences under RE is a shift in the dynamical system: now Y_t is a jump variable, and Y_{t-1} is the only state variable. As a result, now not only are there multiple steady state equilibria under RE, but there are also *multiple solution paths*. As we will see, for any initial state Y_{t-1}, there is a single rational expectations paths to full employment, and and infinite number of paths that lead to Y^L.

Proposition 6. *Under RE and a Taylor rule with $\phi_\pi > 1$, Y^H is a saddle point, thus starting from any employment rate Y_{t-1} in the vacinity of Y^H there is a unique path to full employment.*

Proof. See appendix B. □

Figure 6a, blue line, shows the convergence to full employment for an initial employment rate of $Y_{t-1} = .95$. While there is only a single solution path that leads to Y^H, there are an there are an infinite number of paths to Y^L.

Proposition 7. *Under RE and a Taylor rule with $\phi > 1$, Y^L is a stable sink: there are an infinite number of solution that lead from any initial starting condition Y_{t-1} to Y^L.*

Proof. See appendix B. □

Figure 6a, green line, shows one such path, from a starting value of $Y_{t-1} = .95$.

5.1 Interest rate peg

The first Neo-Fisherian property of this model (*NF-1*) is the ability of an interest rate peg to eliminate the bad steady state.

Proposition 8. *Under an interest rate peg $I_t = R^* \cdot \pi^*$, there is only a single steady state at full employment. The full employment steady state is a stable sink, with an infinite number of paths converging towards full employment.*

Proof. See section B. □

The interest rate peg ensures that the lower steady no longer exists, since in any steady state it must be the case that $\pi = \pi^*$. While this property is somewhat stabilizing, the interest rate peg does introduce another property which is somewhat destabilizing: under a peg the full employment steady state is no longer a saddle point. There are therefore an infinite number of rational expectation

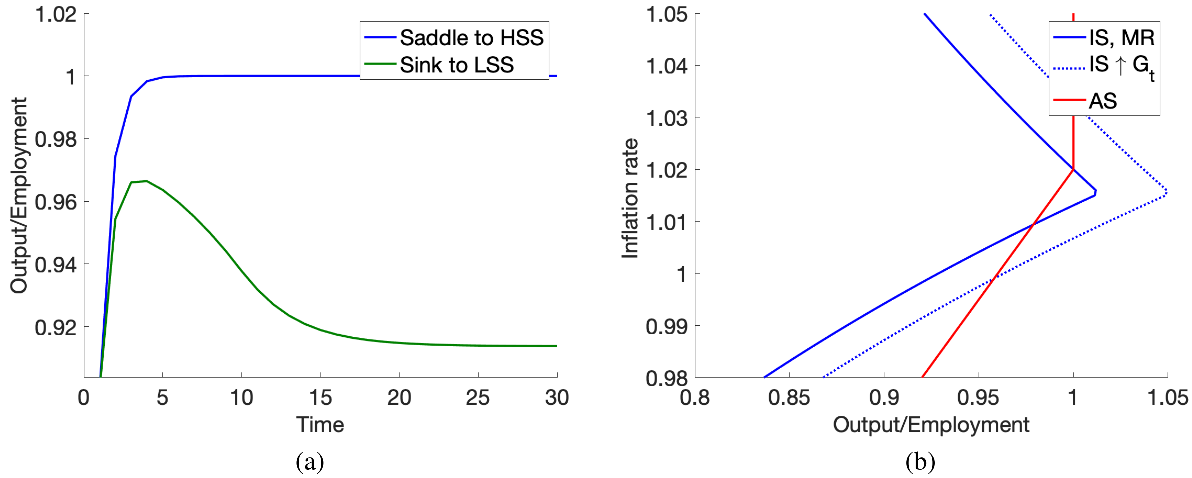


Figure 6: Model with rational expectations. (a) Transition paths: saddle to high steady state, sink to low steady state. (b) IS-Exp under Mertens & Ravn expectations.

paths that lead away from the full employment steady state before eventually returning.

Why does a peg ensure that output always returns to full employment? The stability is analyzed in detail in section 6, but the basic idea is that under rational expectations the IS-Exp equation can be viewed as a pseudo-Euler equation, in which a higher real interest rate leads to a higher growth rate of output.[13] Not only does output grow towards full employment, but the growth is non-explosive. On any increasing path of output, as output increases inflation increases, leading to lower interest rates and lower growth rates of output, preventing output from exploding.

5.2 Fiscal Policy under Mertens and Ravn

Because with rational expectations there are both multiple steady states, and for one of the two steady states multiple solution paths to the steady state, it is not straightforward to examine the effects of fiscal policy on the path of output. Independent of government policy, whatever expectations are the economy can jump anywhere.

Nevertheless, we proceed in studying the effect of government spending with expectations similar to that in Mertens and Ravn (2014). We assume agents are hit with a negative expectation shock, which causes output to be unexpectedly low ($Y_t = Y_t^U$). Output in the future follows a Markov process. With probability z output next period will continue at the same low level $Y_{t+1} = Y_t^U$. With

[13]Of course under the OLG structure in the model there is no aggregate Euler equation which follows directly from a representative agent's optimality condition, but the middle generation is always on its Euler equation.

probability $(1-z)$, expectations will recover, and the economy will return to full employment. A crucial factor in this expectation formation is that the actual level of output in the economy at period t is endogenous, which in turn means expected output in period $t+1$ is endogenous.

The expenditure function is given by

$$Spend_t^{MR}(Y_t^U) = \nu Y_{t-1} + [\overline{MPC}(\widehat{R_t}) + z\chi(\widehat{R_t})]Y_t^U +$$
$$(1-z)\chi(\widehat{R_t})Y^H + \Theta(\tau_t)G_t - \Upsilon(\tau_t)B_t^G + \Upsilon(\tau_t)\frac{B_{t+1}^G}{\widehat{R_t}}$$

The main difference between this model and the model with adaptive expectations is the additional feedback between current income and expenditure: $\frac{\partial Spend_t^{MR}}{\partial Y_t} > \frac{\partial Spend_t}{\partial Y_t}$. This additional feedback means that at the low equilibrium the slope of the expenditure function can be greater than one, leading to Neo-Fisherian properties.

Proposition 9. *If*

- *Expected future output will continue at the current low level with probability z and return to full employment with probability $(1-z)$*
- *The slope of the GE expenditure function is greater than 1: $\frac{\partial Spend_t^{MR}}{\partial Y_t} > 1$*
- *The increase in government spending is tax financed, i.e. there is no change in B_{t+1}^G*

then $\frac{\partial Y_t}{\partial G_t} < 0$, and government spending decreases output.

Figure 6b shows the updated IS-Exp curve and reveals multiple equilibria where the curve intersects with the AS curve. If expectations are high the economy converges on the high equilibrium, but if they are low they move to the unintended equilibrium. The second Neo-Fisherian property of the model (*NF-2*) is apparent from the effect of an increase in government spending, which as usual shifts the IS-Exp curve to the right. But as seen in figure 6b, the rightward shift in the IS-Exp curve lead to a *drop in output* in the low equilibrium. We thus see that under Mertens & Ravn expectations, an increase in government spending or a negative interest shock only makes the bad equilibrium worse.

5.3 Fiscal policy under alternative expectations

Under rational expectations there are an infinite number of equilibria, of which the Mertens & Ravn type expectations are only one particular form. We now explore the effect of fiscal policy under several different types of expectations, and see that the *NF-2* results above do no generalize to all expectations.

We make one small modifications to the expectation setup: instead of having expectations in the low state endogenous to this period's output, we assume it is a fixed level. In particular, agents expect that output next period will be some *fixed level* $\overline{Y^U}$ with probability z or will recover to full employment with probability $(1-z)$. Under this assumption, expectations are not longer endogenous, and there is no feedback from current output to expectations of future output. Under this assumption the marginal propensity to consume is the same as in the model under adaptive expectations, and there are no Neo-Fisherian properties.

The one exception to the multiplcitiy present under rational expectations is the unique saddle path that exists to Y^H, thus in one sense it is most natural to study the effects of fiscal and monetary policy on the saddle path. Along the saddle path there are no Neo-Fisherian properties of government spending.

Proposition 10. *Along the saddle path transition to the high steady state, a tax financed increase in government spending increases output in all periods:*

$$\frac{\partial Y_{t+s}}{\partial G_t} \geq 0, \; s \geq 0$$

Proof. See section appendix B. □

Figure 7a, dark blue line, shows the saddle paths for the system without government spending, and the green line shows the system with a one time tax financed increase. A temporary shock to government spending in period t shifts output and employment along the entire path. In this context, fiscal and monetary policy *does not* possess Neo-Fisherian properties.[14]

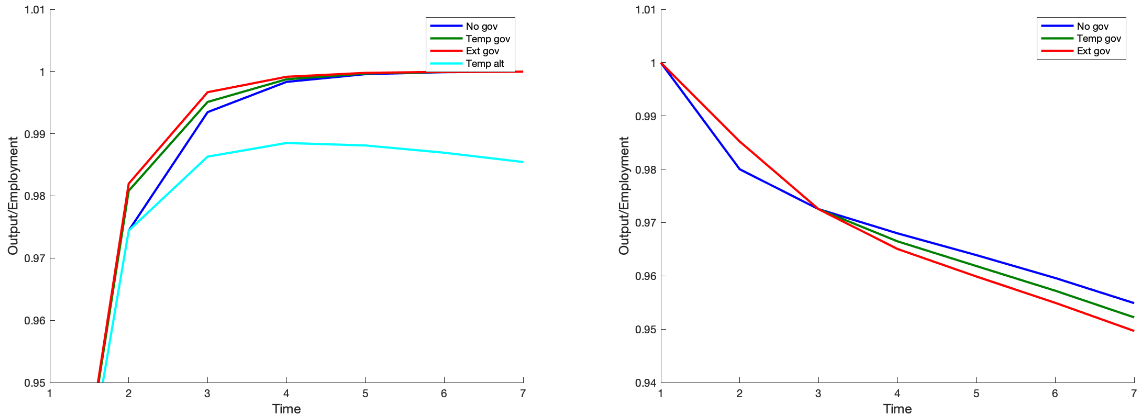
6 Searching for Neo-Fisher

In the model with rational expectations, the effects of fiscal and monetary policy have the opposite signs of what is traditionally found in Keynesian models. One way to understand these un-Keynesian effects is through an analysis of the Keynesian cross. We return to the equilibrium condition of the model, that income is equal to spending:

$$\begin{aligned} Y_t = \overline{MPC}(\widehat{R_t})Y_t + \nu Y_{t-1} + \chi(\widehat{R_t})\widehat{Y_{t+1}} \\ +\Theta(\tau_t)G_t - \Upsilon(\tau_t)B_t^G + \Upsilon(\tau_t)\frac{B_{t+1}^G}{\widehat{R_t}} \end{aligned} \tag{17}$$

Figure 8a displays equation 17 holding $\widehat{R_t}$ and $\widehat{Y_{t+1}}$ fixed, what is denoted a *partial equilibrium Keynesian cross*, and shows a standard pattern: a slope of the aggregate spending curve that is less than one. In partial equilibrium, increased

[14] Along the 'sink' path to Y^L, it is difficult to study the effect of government spending because of the multiplicity of equilibrium paths. However, if we fix the current state of the economy Y_{t-1} and expectations of the future, Y_{t+1}, we can proceed. Figure 7b shows the effects of government spending under these conditions. Output is fixed at time 0 and 2, and the system with no government spending is shown in the blue line. A temporary increase in government spending increases output in period 1, but lowers output after period 2.



(a) (b)

Figure 7: (a) Saddle paths to Y^H with government spending (b) Sink paths to Y^L with government spending.

fiscal spending or a cut in interest rates will shift the curve upwards, leading to an increase in output. But this will not necessarily hold in general equilibrium.

If we allow interest rates and expectations to change as income increases, the shape of the aggregate spending curve will shift. Figure 8b represents a *general equilibrium* Keynesian cross under M&R expectations. The curve is general equilibrium in two ways. First, the real interest rate is allowed to change to the value it would actually take in equilibrium if output was at Y_t. This involves plugging in the monetary policy and aggregate supply curve into equation 17 to make real interest rates a function of Y_t.[15] The second change from the PE version is that expectations become a function of Y_t. Under Mertens & Ravn expectations, $\widehat{Y_{t+1}} = zY_t^U + (1-z)Y^H$: as output increases, expectations of future output increase.

These two changes create two additional feedback loops from income to spending. First, when the economy is at the ZLB, an increase in income will raise inflation, boosting spending. Second, an increase in income will increase expectations of income next period, also boosting spending.

If the feedback from income to spending is strong enough, the *slope of the Keynesian expenditure function can be greater than one*, as seen in figure 8b. Along certain points of the support the slope is greater than 1, and the expen-

[15]To calculate this value, we start with the Fisher equation, $\widehat{R_t} = I_t/\pi_t$, and plug in the monetary policy rule $\widehat{R_t} = \frac{max(1,(1+i^*)(\frac{\pi_t}{\pi^*})^{\phi_\pi})+\epsilon_t^i}{\pi_t}$. To determine the inflation rate that would be present in equilibrium, we plug in the aggregate supply curve for π_t: $\pi_t = \pi^* - \kappa(1-Y_t)$, yielding

$$\widehat{R_t}(Y_t) = \frac{max(1,(1+i^*)(\frac{(\pi^*-\kappa(1-Y_t))}{\pi^*})^{\phi_\pi})+\epsilon_t^i}{\pi^* - \kappa(1-Y_t)}.$$

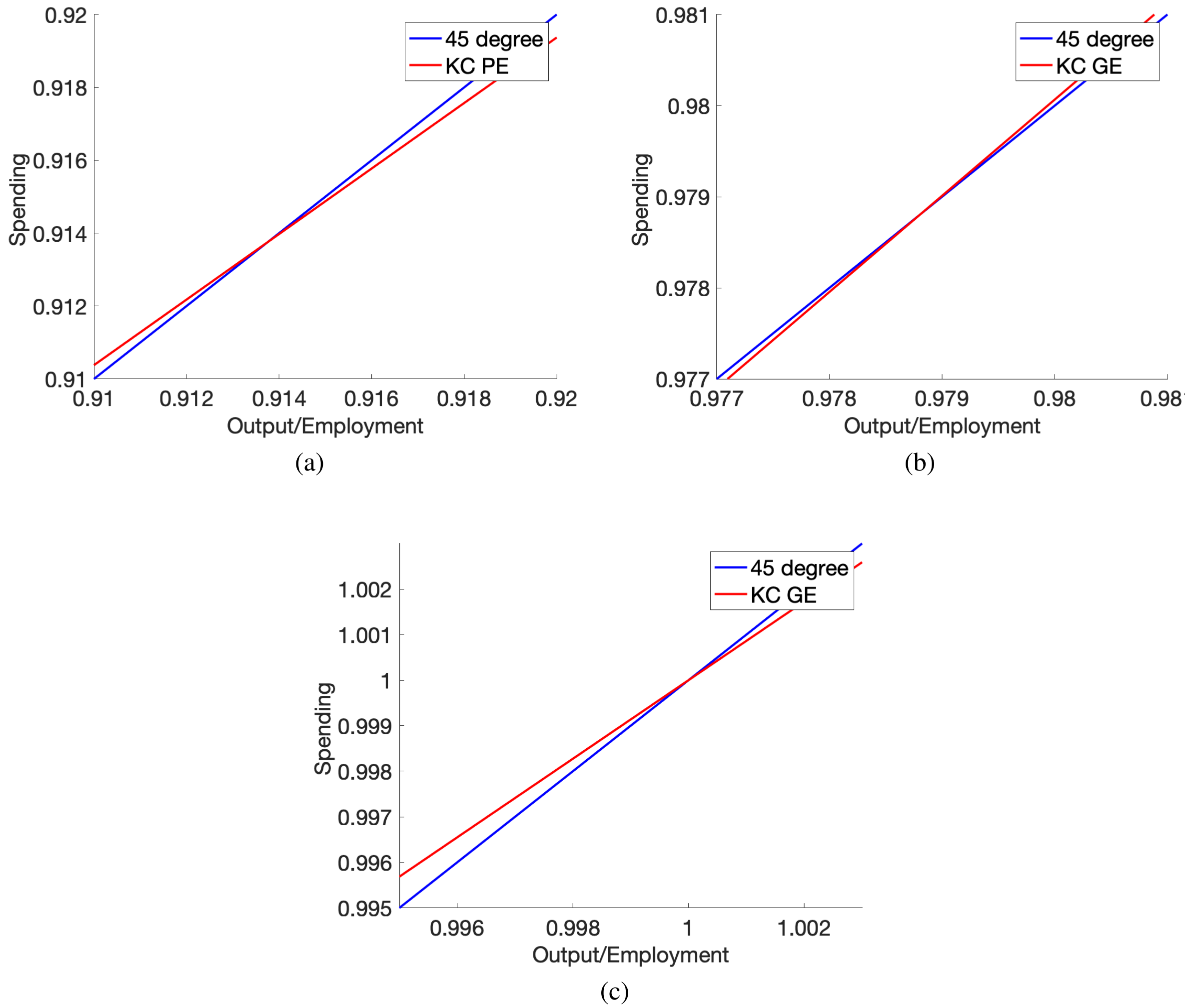


Figure 8: Mertens & Ravens expectations (a) Partial equilibrium Keynesian cross (b) general equilibrium Keynesian cross, z=.9, zoomed in around Y^U (c) general equilibrium Keynesian cross, z=.1

diture function intersects the 45 degree line from below. In this region, all of the usual comparative statics are reversed for the lower equilibrium. Now when the aggregate spending curve shifts up, output at the lower steady state Y_t^U will decrease.

Thus one way to characterize the NF-2 results is as follows: NF-2 results will ensure if and only if, once general equilibrium effects are taken into account, spending moves more than one-to-one with current income.

If we decrease the feedback in the M&R model, the slope of the general equilibrium Keynesian cross decreases, and the Neo-Fisherian properties go away. This is shown in figure 8c.

6.1 Neo-Fisher with non rational expectations

The above section raises the possibility that *NF-2* results could emerge even under non-rational expectations, if an additional feedback loop between current output and expectations of output is introduced. This would in turn increase the slope of the GE-Keynesian expenditure function to be greater than 1, and usher in the Neo-Fisherian results.

We now modify belief formation to allow expected output $\widehat{Y_{t+1}}$ to be a function of current output Y_t:

$$\text{Adaptive: } \widehat{Y_{t+1}} = \lambda Y_t + (1-\lambda)\widehat{Y_t} + \epsilon_t^e \tag{18}$$

$$\text{Extrapolative: } \widehat{Y_{t+1}} = Y_t + \xi(Y_t - Y_{t-1}) + \epsilon_t^e \tag{19}$$

We focus the exposition on extrapolative expectations, since it is easier to increase feedback; the ξ parameter can be greater than 1, while traditional adaptive expectations has $\lambda < 1$. The Keynesian expenditure function is given by

$$\begin{aligned}\text{Spend}_t(Y_t) = (\nu - \xi\chi(\widehat{R_t}))Y_{t-1} + \Theta(\tau_t)G_t - \Upsilon(\tau_t)B_t^G + \Upsilon(\tau_t)\frac{B_{t+1}^G}{\widehat{R_t}} + \\ \left[\overline{MPC}(\widehat{R_t}) + \chi(\widehat{R_t})(1+\xi)\right] Y_t\end{aligned}$$

As ξ[16] increases, there is more feedback between current income and expectations of future income. This increases the GE MPS of agents, since every additional dollar of income increases expectations more. If ξ increases enough, the MPS will be greater than 1 and *NF-2* results will ensue.

Figure 9a shows transition paths, comparing the effect of government spending when feedback is high ($\xi = .5$) compared to when feedback is low ($\xi = .1$). The red line shows the $\xi = .5$ system without government spending, which converges to Y^L. There is a one time positive shock to government spending in period 2, which shifts output down along the transition path, showing Neo-Fisherian properties. When $\xi = .1$ (blue lines), the system converges to Y^H and government multipliers have standard signs.

[16] Or λ under adaptive expectations.

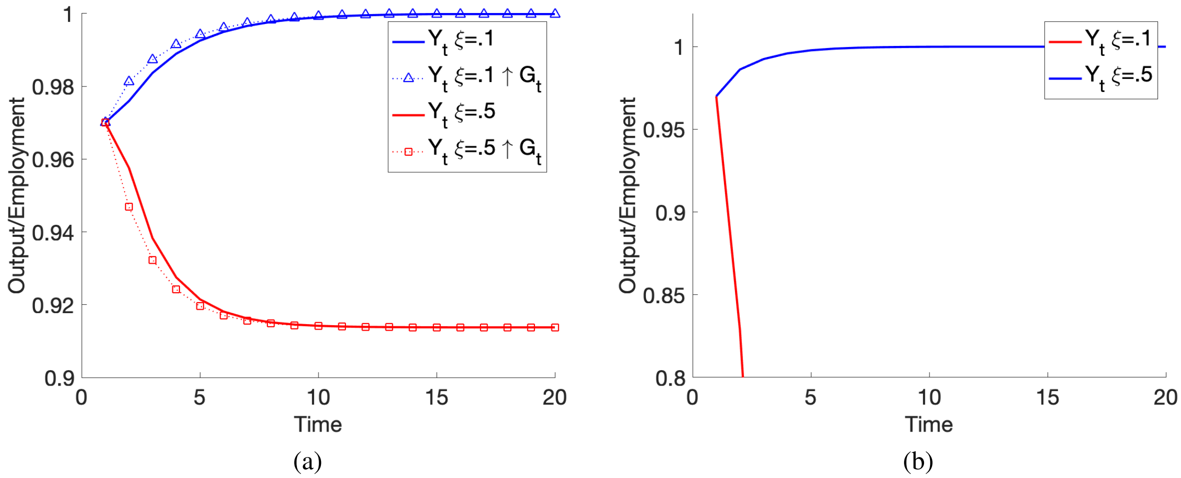


Figure 9: Extrapolative expectations based on current output, $\xi = .5$ vs $\xi = .1$ (a) Transition paths with and without government spending (b) transition paths with interest rate peg.

6.2 Stability of the interest rate peg

Under rational expectations, section 5.1 showed that Y^H is stable under an interest rate peg, in the sense that if the economy is ever in a recession, a peg will ensure that the economy always returns to full employment. This is a surprising result because apriori it would seem that the higher real interest rates of a peg would tend to destabilize the economy and cause a deeper recession.

To better understand this result, we return to the IS-Exp equation of the model, rearranging to write Y_{t+1} as a function of Y_t:

$$Y_{t+1} = \frac{\widehat{R_t}}{(1-\gamma)mpc^m} \times \left(\frac{Y_t}{\alpha(\widehat{R_t})} - \nu Y_{t-1} - \Theta(\tau_t)G_t + \Upsilon(\tau_t)B_t^G - \Upsilon(\tau_t)\frac{B_{t+1}^G}{\widehat{R_t}} \right) \tag{20}$$

Equation 20 displays a pseudo Euler equation; a higher real interest rate raises the growth rate of output. When output is below target, a peg ensures a high growth rate of output. As output approaches full employment, high inflation pushes down real interest rates, leading output to stabilize at the target.

Equation 20 shows one relationship between rational expectations and the *NF-1* result. Rational expectations ensures Y_{t+1} is present in equation 20, and thus the dynamics of the economy are governed by the forward looking Euler equation.

Under non-rational expectations, the Euler equation no longer completely determines the dynamics of output; the dynamics of expectations also contribute to the dynamics of output. Section 4.2 shows that an interest peg is not stabilizing with non-rational expectations that depend upon past output. The reason being is that now in equation 20 Y_{t+1} is replaced with $\widehat{Y_{t+1}}$, and there is no longer a simple positive relationship between real interest rates and the growth rate of output. Instead, the relationship is more complex; higher real interest rates in period t will decrease output in period t, which in turn will further push down output in the future due to lower past output and lower future expectations.

We now show the stability of a peg can be restored, however, if we can increase the feedback loop between current and future income. Section 6.1 showed that if the MPS is greater than 1, the normal signs of fiscal and monetary policy are reversed. When this occurs, a rise in real interest rates raises output, expectations of output, and output growth. The reason why a higher interest rate increases output growth, it should be noted, is through very different channels than under rational expectations.

Figure 9b shows transition paths under an interest rate peg for $\xi = .5$, showing a convergence to Y^H. If the feedback is too low, with $\xi = .1$, there are no Neo-Fisherian properties, and an interest rate peg causes the economy to converge to Y^0.

6.3 Properties of Neo-Fisherian states

In section 6 it was shown that Neo-Fisherian results occur once the general equilibrium MPC is greater than 1. But as the multiplier is equal to $1/(1-\text{GE MPC})$, the Neo-Fisherian region can only begin after a singularity point in which the multiplier is $-\infty$.[17] Under Mertens & Ravens expectations, if z is too small there is no Neo-Fisherian equilibrium. But for a z high enough such that $NF-2$ just holds, the multiplier is $-\infty$. As z increases, the MPC increases the magnitude of the multiplier decreases. This is depicted in figure 11a; at this point, multipliers are negative infinity. As z increases, the magnitude of the negative multipliers decreases, as depicted in figure 11b.

For the model under extrapolative expectations based on current income, for ξ small enough, there will be standard multipliers. An increase in ξ will initially raise multipliers, and as ξ increases the multiplier will approach ∞. At this singularity, the multiplier crosses over to $-\infty$, which is the point at which Neo-Fisherian results begin. Figure 10 shows mutlipliers as a function of ξ, and shows the point at which multipliers discontinuously jump from ∞ to $-\infty$. It is only after this point is reached that Neo-Fisherian results will hold.

6.4 Aggregate Demand Expectations Curve

There is one more perverse property of the model under Neo-Fisherian conditions. Under Neo-Fisherian states, whenever there is a positive shock to expectations ϵ_t^e, in equilibrium expectations $\widehat{Y_{t+1}}$ actually *decrease*. The reason why

[17] Bilbiie (2018*b*) shows a result that is similar in spirit but in a slightly different context, where the region between Neo-Fisherian and Keynesian regions is an inflection point.

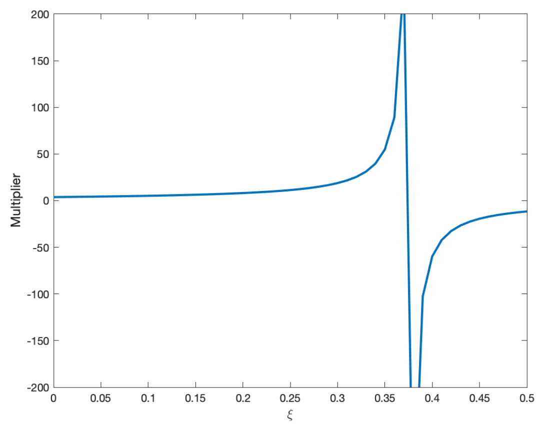
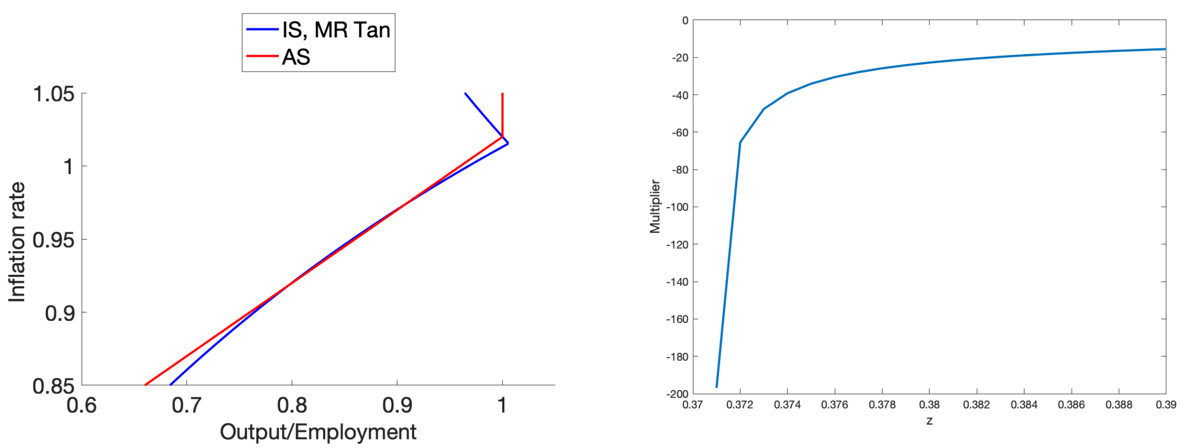


Figure 10: Government multipliers as a function of ξ, adaptive expectations.



(a) (b)

Figure 11: Mertens & Ravn expectations (a) IS-Exp curve: government multipliers are $-\infty$ (b) multipliers as a function of z.

is because a positive shock to expectations under Neo-Fisherian conditions decreases current output Y_t, which transfers into even lower expectations. Under a Neo-Fisherian model, the best way to solve a problem of a low-expectation equilibrium is to lower everyone's expectations even further.

To show this, we will characterize the equilibrium with a change in variables. We first derive an aggregate demand expectations curve (AD-Exp), which is in the $\{Y_t, \widehat{Y_{t+1}}\}$ space. This curve is derived from the IS-Exp curve, equation 16. We fix different expectations for $\widehat{Y_{t+1}}$, and for each expectation we find the intersection of the IS-Exp and AS curves, yielding equilibrium Y_t. This gives a curve of equilibrium output Y_t as a function of expectations of future output, $\widehat{Y_{t+1}}$. Figure 2b traces out two points of this aggregate demand curve under low and high expectations. This AD-Exp curve is shown in figure 12a, blue line.

The second curve is the "belief formation" (Bel) curve, which shows how expectations of output change under different levels of current income, and is derived from equation 19,. Under extrapolative expectations, the slope of the curve is $\frac{1}{1+\xi}$. This is displayed by the green line in figure 12a.

Equilibrium in the model occurs at the intersection of the AD-Exp and Bel curves. At this point, markets clear, and future beliefs are consistent with the current level of output which generate these beliefs.

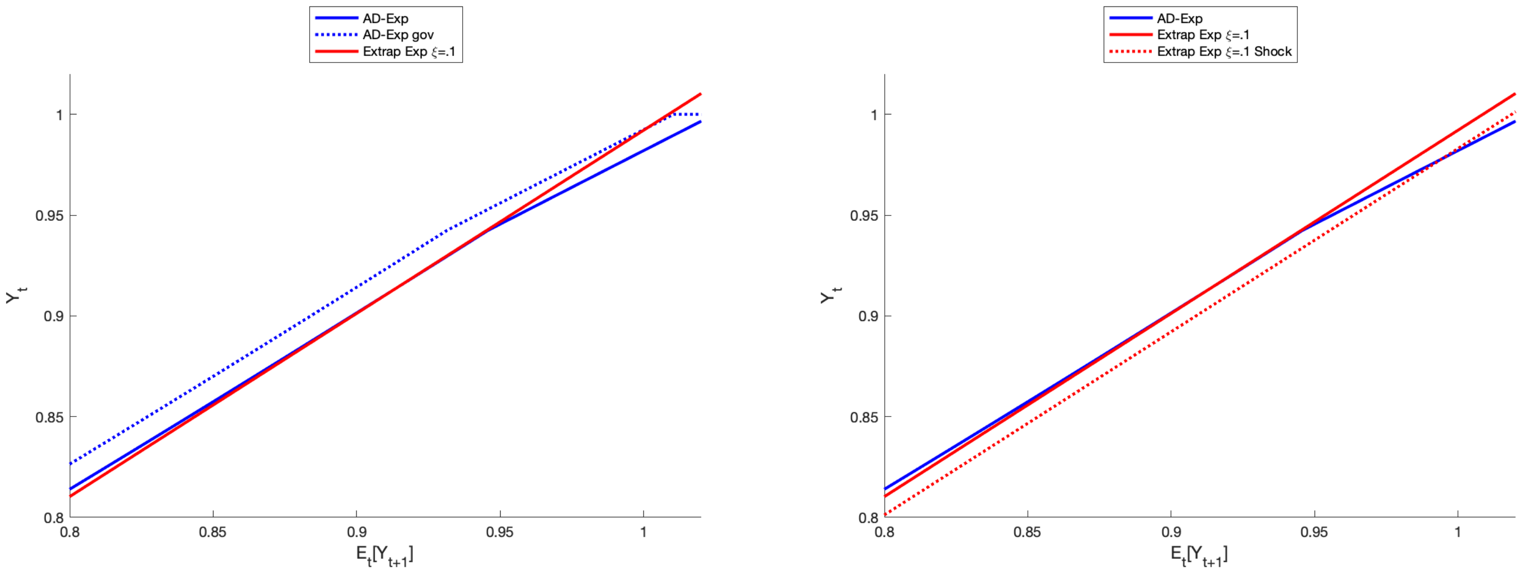
Whether the model has Neo-Fisherian properties depends on the relative slopes of the AD-Exp and Bel curves. In figure 12a, the slope of the AD-Exp is less steep than than the slope of the Bel curve, and there are no Neo-Fisherian properties. Figure 12a shows the effect of an increase in government spending: the AD-Exp curve shifts up, leading to an increase in both expectations and output. Figure 12b shows the effect of a positive shock to expectations ϵ_t^e: the Bel curve shifts to the right, increasing both output and expectations.

Geometrically, it is clear that if slope of the AD-exp is less steep than the slope of the Bel curve, there are both no Neo-Fisherian properties, *and* a positive shock to beliefs leads to an increase in expectations in equilibrium. It is equally clear that that if the slope of the AD-Exp curve is steeper than the Bel curve, there will be both Neo-Fisherian properties and a positive shock to beliefs will lead to a decline in expectations.

In figure 13a, we set $\xi = .5$, which lowers the slope of the Bel formation curve below that of the AD-Exp curve, and heralds the return of Neo-Fisherian properties. An increase in government spending in figure 13a now *lowers* output and expectations of future output. Figure 13b shows the effect of a positive shock to expectations, which leads to *lower* expectations and output.

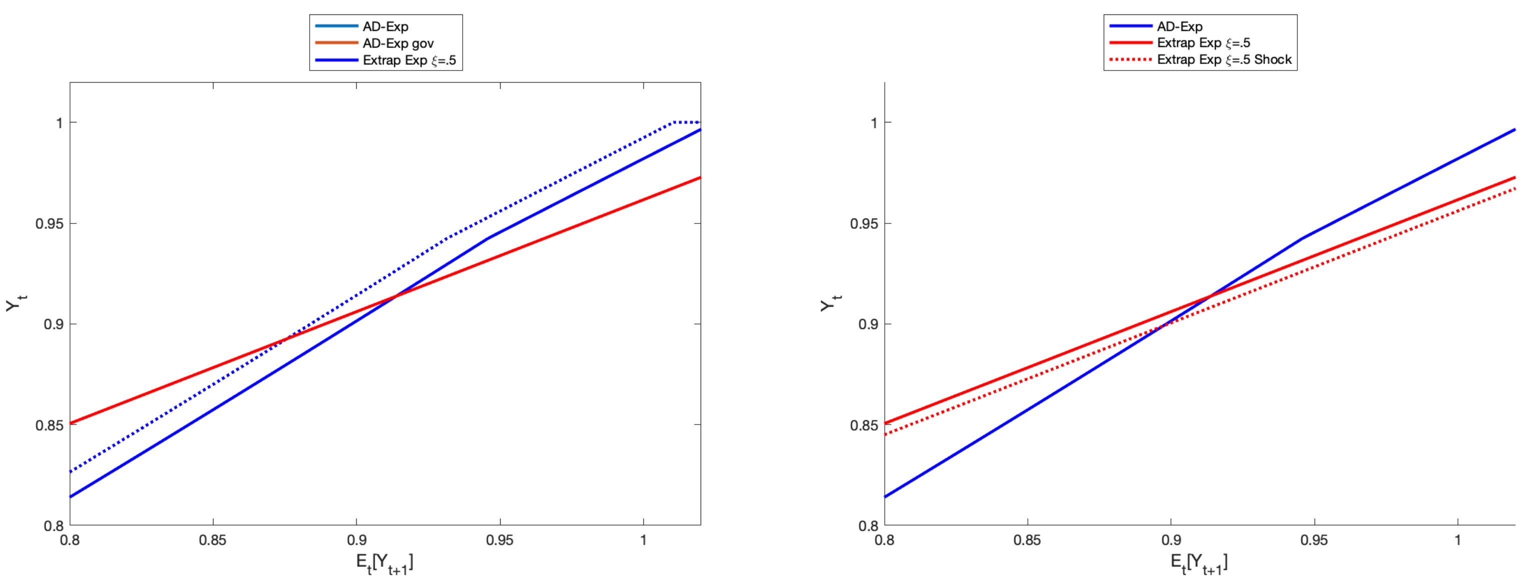
7 Discussion and conclusion

The theoretical analysis shows that negative expectations about future output and employment can lead to recessions. These can occur even in the absence of price stickiness and the ZLB. This has important policy implications —- if the primary cause of recessions is not price stickiness or the ZLB, it is less important



(a) (b)

Figure 12: Aggregate demand extrapolative expectations equilibrium, $\xi = .1$. No Neo-Fisherian properties.



(a) (b)

Figure 13: Aggregate demand extrapolative expectations equilibrium, $\xi = .5$. Neo-Fisherian properties.

to devise policies to increase wage flexibility or getting around the ZLB.[18] This is not an argument that wage stickiness or the ZLB doesn't exist, but rather that they may be the primary factors causing unemployment in recessions.

A second conclusion from this model is the importance of expectations as an independent force in macroeconomics. This has long been stressed by the sunspot literature, who focus on rational expectations equilibria. The recent research innovation of asking people their beliefs about the future (see for example, the Survey of Consumer Expectations Armantier et al. (2017)) provide evidence that expectations are unlikely to be "rational" in the Lucas sense — the sheer diversity of believes about inflation, housing prices, and unemployment is itself compelling evidence. More research needs to be done about how beliefs are formed — for example, in Robert Shiller's recent work on "narrative economics" (Shiller (2017)).

The results of this paper show that the assumption of rational expectations is not innocuous when studying model of recessions. Neo-Fisherian results are related to relational expectations because of the large degree of feedback between income and expectations in these models. However, this paper does show that rational expectations are not a necessary condition for NF results, and that all is needed is a sufficient degree of feedback between income and expectations.

A final takeaway from this research is the dangers of instantaneous multipliers. Keynesians that live by the sword with large government multipliers will die by the sword under Neo-Fisherian results. Work remains to be done to unpack the flow between spending, income, and expectations.

[18]Previous research found that increased price flexibility at the ZLB can increases the severity of recessions (the "paradox of flexibility" of Eggertsson and Krugman (2012), and also found in Eggertsson, Mehrotra and Robbins (2019)).

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Online Appendix for
Expectation driven recessions

Jacob A. Robbins

A Full derivation of model

Consumers maximize

$$U_t = \log C_t^m + \beta \log \widehat{C_{t+1}^o}$$

subject to budget constraints

$$C_t^m + B_{t+1}^o/\widehat{R_t} = \gamma w_t L_t + \gamma \Pi_t + B_t^m = \gamma Y_t + B_t^m \tag{A.1}$$

$$C_t^o = (1-\gamma) w_t L_t + (1-\gamma)\Pi_t + B_t^o = (1-\gamma) Y_t + B_t^o, \tag{A.2}$$

where B_{t+1} is the face value of bonds purchased at time t, at a price $1/\widehat{R_t}$.

Young agent is constrained, and thus $C_t^y = D * Y_t/\widehat{R_t}$. Middle agent is on Euler equation, yielding the first order condition $\widehat{C_{t+1}^o} = \beta \widehat{R_t} C_t^m$. Plugging this into the budget equation yields the optimal consumption

$$C_t^m = \frac{1}{1+\beta}(\gamma Y_t + B_t^m + (1-\gamma)\widehat{Y_{t+1}}/\widehat{R_t}) \tag{A.3}$$

The old agent simply consumers all of his bonds and assets like there is no tomorrow, and thus

$$C_t^o = (1-\gamma) Y_t + B_t^o \tag{A.4}$$

We can eliminate the asset holdings in all of these optimal consumption equations. In equilibrium, middle age bond holdings are simply what was borrowed when the agent is young, and thus $B_t^m = D * Y_{t-1}$. In addition, old age bond holdings are simply what was lent when the agent is middle aged. Since the middle aged agent always lends to the young, we have $B_t^o = D * Y_{t-1}$.

Optimal consumption for each generation is thus

$$C_t^y = D * Y_t/\widehat{R_t}$$

$$C_t^m = \frac{1}{1+\beta}(\gamma Y_t - D * Y_{t-1} + (1-\gamma)\widehat{Y_{t+1}}/\widehat{R_t})$$

$$C_t^o = D * Y_{t-1} + (1-\gamma) Y_t$$

and aggregate consumption is

$$Y_t = C_t^y + C_t^m + C_t^o =$$

$$D * Y_t/\widehat{R_t} + \frac{1}{1+\beta}(\gamma Y_t - D * Y_{t-1} + (1-\gamma)\widehat{Y_{t+1}}/\widehat{R_t}) + D * Y_{t-1} + (1-\gamma) Y_t$$

Grouping terms,

$$Y_t = [\frac{\beta}{1+\beta}D]Y_{t-1} + [D/\widehat{R_t} + \frac{1}{1+\beta}\gamma + (1-\gamma)]Y_t + [\frac{1}{1+\beta}\frac{(1-\gamma)}{\widehat{R_t}}]\widehat{Y_{t+1}}$$

Defining $\nu = \frac{\beta D}{1+\beta}$, $\chi(\widehat{R_t}) = mpc^m(1-\gamma)/\widehat{R_t}$, and

$$\overline{MPC}(\widehat{R_t}) = [D/\widehat{R_t} + \frac{1}{1+\beta}\gamma + (1-\gamma)],$$

we have

$$Y_t = \nu Y_{t-1} + \overline{MPC}(\widehat{R_t})Y_t + \chi(\widehat{R_t})\widehat{Y_{t+1}} \tag{A.5}$$

Solving for Y_t yields the Investment-Savings-Expectations (IS-Exp) curve.

$$Y_t = \alpha(\widehat{R_t})\left(\nu Y_{t-1} + \chi(\widehat{R_t})\widehat{Y_{t+1}}\right) \tag{A.6}$$

The traditional Keynesian multiplier $\alpha(\widehat{R_t}) = \frac{1}{\left[1-\frac{D}{\widehat{R_t}}-\frac{1}{1+\beta}\gamma-(1-\gamma)\right]}$

With government spending and taxation, spending is affected by two contrasting forces: the increase in spending from the federal government, and the decrease in spending from individuals from increased taxation. With taxation, optimal consumption for each generation (assuming no old age taxation) is given by

$$\begin{aligned} C_t^y &= D * Y_t/\widehat{R_t} - T_t^Y \\ C_t^m &= \frac{1}{1+\beta}(\gamma Y_t - D * Y_{t-1} - T_t^m + (1-\gamma)\widehat{Y_{t+1}}/\widehat{R_t}) \\ C_t^o &= D * Y_{t-1} + (1-\gamma)Y_t \end{aligned}$$

We note that the marginal propensities to consume out of past, current, and future income are unchanged. The only differences in the IS-Exp curve will be two additive terms: the direct effects of government spending on consumption, and the indirect effects of taxation.

We focus on a fiscal policy in which a fraction of the total taxes is paid by each generation to balance the the budget: τ_t^y and τ_t^m. Given the government budget constraint, the taxes of each generation g equal to $T_t^g = \tau_t^g * (B_t^G + G_t - \frac{B_{t+1}^G}{\widehat{R_t}})$. The total drag on spending will equal the sum of the effect on the young and middle age generation: $\tau_t^y * (B_t^G + G_t - \frac{B_{t+1}^G}{\widehat{R_t}}) + \frac{1}{1+\beta}\tau_t^m * (B_t^G + G_t - \frac{B_{t+1}^G}{\widehat{R_t}})$. Defining $\Upsilon(\tau_t) = \tau_t^y + \frac{1}{1+\beta}\tau_t^m$, $\Theta(\tau_t) = 1 - \tau_t^y - \frac{1}{1+\beta}\tau_t^m$, the total effect on consumption is given by $\Theta(\tau_t)G_t - \Upsilon(\tau_t)B_t^G + \Upsilon(\tau_t)B_{t+1}^G$.

The IS-Exp curve is then given by

$$\begin{aligned} Y_t = \alpha(\widehat{R_t})\Big(&\nu Y_{t-1} + \chi(\widehat{R_t})\widehat{Y_{t+1}} + \\ &\Theta(\tau_t)G_t - \Upsilon(\tau_t)B_t^G + \Upsilon(\tau_t)\frac{B_{t+1}^G}{\widehat{R_t}}\Big) \end{aligned} \tag{A.7}$$

In a steady state in which spending as a fraction of output is given by g and government debt as a fraction of output by b^g, the steady state real interest rate is given by

$$R = \frac{(1-\gamma) + D(1+\beta) + \Upsilon b^g(1+\beta)}{\beta(\gamma - D) + (\Upsilon b^g - \Theta g)(1+\beta)}$$

B Proofs

B.1 Preliminary calculations

An expression for $1 - \overline{MPC}(\widehat{R_t})$:

$$1 - \overline{MPC}(\widehat{R_t}) = \frac{\gamma\beta\widehat{R_t} - D(1+\beta)}{(1+\beta)\widehat{R_t}} \tag{A.8}$$

The derivative of MPC with respect to $\widehat{R_t}$ is

$$\frac{\partial\overline{MPC}}{\partial\widehat{R_t}} = (-1)D\widehat{R_t}^{-2} < 0$$

In a steady state, we have

$$Y = \alpha(R^*)\left(\nu Y + \chi(R^*)Y\right) = \alpha(R^*)(\nu + \chi(R^*))Y$$

Therefore we have

$$\alpha(R^*)(\nu + \chi(R^*)) = 1 \tag{A.9}$$

$$1 - \overline{MPC}(R^*) = \nu + \chi(R^*) \tag{A.10}$$

From the Fisher equation,

$$\widehat{R_t} = \frac{I_t}{\pi_t} \Rightarrow \tag{A.11}$$

$$\frac{\partial\widehat{R_t}}{\partial\pi_t} = -\pi^{-2}I_t + \frac{\partial I_t}{\partial\pi_t}\pi_t^{-1} \tag{A.12}$$

Then from Taylor rules

$$\frac{\partial I_t}{\partial\pi_t} = \begin{cases} \phi_\pi\pi_t^{-1}[I_t - \epsilon_t^i] & \text{if ZLB is not binding} \\ 0 & \text{if ZLB is binding} \end{cases}$$

We thus have

$$\frac{\partial\widehat{R_t}}{\partial\pi_t} = \begin{cases} \pi_t^{-2}I_t(\phi_\pi - 1 - \epsilon_t^i/I_t) > 0 & \text{if ZLB is not binding} \\ -\pi_t^{-2}I_t < 0 & \text{if ZLB is binding} \end{cases}$$

The Phillips curve is given by $\pi_t = \pi^* - \kappa(1 - Y_t)$, thus $\frac{\partial\pi_t}{\partial Y_t} = \kappa$. Since we have $\widehat{R_t}(Y_t) = f(\pi_t) = f(\pi_t(Y_t))$, we have $\widehat{R_t} = h(Y_t)$. We thus have

$$\frac{\partial\widehat{R_t}}{\partial Y_t} = \frac{\partial\widehat{R_t}}{\partial\pi_t}\frac{\partial\pi_t}{\partial Y_t} = \begin{cases} \kappa\pi_t^{-2}I_t(\phi_\pi - 1 - \epsilon_t^i/I_t) > 0 & \text{if ZLB is not binding} \\ -\kappa\pi_t^{-2}I_t < 0 & \text{if ZLB is binding} \end{cases}$$

B.2 Propositions

Proof of proposition 1.

Proof. This follows from an analysis of the IS-Exp curve. As long as the GE MPS is less than 1, multipliers will have normal signs. From the IS-Exp we define

$$F(Y_t, \epsilon_t^e) = \nu Y_{t-1} + \overline{MPC}(\widehat{R_t}(Y_t))Y_t + \chi(\widehat{R_t})\widehat{Y_{t+1}} - Y_t = 0 \tag{A.13}$$

From the implicit function theorem,

$$\frac{\partial Y_t}{\partial \epsilon_t^e} = -\frac{\frac{\partial F}{\partial \epsilon_t^e}}{\frac{\partial F}{\partial Y_t}}$$

For this derivative to be positive, the denominator must be negative, since the numerator is positive:

$$\frac{\partial F}{\partial \epsilon_t^e} = mpc^m(1-\gamma)/\widehat{R_t} > 0$$

The denominator is given by

$$\frac{\partial F}{\partial Y_t} = \frac{\partial \overline{MPC}}{\partial \widehat{R_t}}\frac{\partial \widehat{R_t}}{\partial Y_t}Y_t + \overline{MPC}(\widehat{R_t}(Y_t)) - \widehat{R_t}^{-1}\chi(\widehat{R_t})\widehat{Y_{t+1}}\frac{\partial \widehat{R_t}}{\partial Y_t} - 1$$

We note that the condition the denominator is negative is the condition that the general equilibrium Keynesian cross has a slope of less than 1.

The derivative will depend upon whether the ZLB is binding or not. In the case where the ZLB is not binding,

$$\frac{\partial F}{\partial Y_t} = \underbrace{\frac{\partial \overline{MPC}}{\partial \widehat{R_t}}\frac{\partial \widehat{R_t}}{\partial Y_t}Y_t}_{-} + \underbrace{\overline{MPC}(\widehat{R_t}(Y_t))}_{+,<1} \underbrace{-\widehat{R_t}^{-1}\chi(\widehat{R_t})\widehat{Y_{t+1}}\frac{\partial \widehat{R_t}}{\partial Y_t}}_{-} -1 < 0$$

It's more complicate in the case where the ZLB is binding. In this case there is an additional feedback mechanism between income and spending. When the ZLB is binding, higher income means higher inflation, which means lower interest rates. This increases the MPC of the young agent (first term). The lower interest rates also increase the PDV of income for the middle aged agent, causing an increase in spending (third term). Evaluating the derivatives, we have

$$\begin{aligned}\frac{\partial F}{\partial Y_t} &= D\kappa Y_t + \overline{MPC}(\widehat{R_t}(Y_t)) + mpc^m(1-\gamma)\widehat{Y_{t+1}}\kappa - 1 < 0 \\ &\Rightarrow (1-\overline{MPC}(\widehat{R_t}(Y_t))) - D\kappa Y_t - mpc^m(1-\gamma)\widehat{Y_{t+1}}\kappa > 0 \end{aligned}\tag{A.14}$$

If we make the assumption that $\widehat{Y_{t+1}} < 1$, $Y_t < 1$ and $R_t > R^*$, then $(1 - \overline{MPC}(\widehat{R_t}(Y_t))) > (1 - \overline{MPC}(R^*))$, then the conditions becomes

$$(1-\overline{MPC}(R^*)) - D\kappa - mpc^m(1-\gamma)\kappa > 0. \tag{A.15}$$

But at R^*, from equation A.10 we have $(1 - \overline{MPC}(R^*)) = \nu + \chi(R^*)$. So the argument will hold if

$$\frac{\beta D}{1+\beta} + \frac{(1-\gamma)}{(1+\beta)R^*} > D\kappa + \frac{1}{1+\beta}(1-\gamma)\kappa \tag{A.16}$$

This will be the case if $\kappa < \frac{\beta}{1+\beta}$ and $\kappa < \frac{1}{R^*}$ □

Proof of proposition 2.

Proof. From the IS-Exp equation, the dynamical system can be written

$$\begin{aligned} Y_t = \nu Y_{t-1} + \overline{MPC}(\widehat{R_t})Y_t + \chi(\widehat{R_t})\widehat{Y_{t+1}} \\ \widehat{Y_{t+1}} = \lambda Y_{t-1} + (1-\lambda)\widehat{Y}_t + \epsilon_t^e \end{aligned}$$

We linearize the dynamical system around the high steady state

$$\frac{\partial}{\partial Y_t} = \frac{\partial \overline{MPC}}{\partial \widehat{R_t}} \frac{\partial \widehat{R_t}}{\partial Y_t} Y_t + \overline{MPC}(\widehat{R_t}(Y_t)) - \widehat{R_t}^{-1} \chi(\widehat{R_t})\widehat{Y_{t+1}} \frac{\partial \widehat{R_t}}{\partial Y_t}$$

$$\begin{aligned} \tilde{Y}_t = \left[\frac{\partial \overline{MPC}}{\partial \widehat{R_t}} \frac{\partial \widehat{R_t}}{\partial Y_t} Y^H + \overline{MPC}(\widehat{R_t}(Y_t)) - \widehat{R^*}^{-1} \chi(\widehat{R^*}) Y^H \frac{\partial \widehat{R_t}}{\partial Y_t} \right] \tilde{Y}_t \\ + [\nu + \chi(\widehat{R^*})\ \lambda]\widetilde{Y_{t-1}} + \chi(\widehat{R^*})(1-\lambda)\tilde{\widehat{Y}}_t \end{aligned}$$

We define the GE Keynesian multiplier as

$$a_1 = \left[1 - \left[\frac{\partial \overline{MPC}}{\partial \widehat{R_t}} \frac{\partial \widehat{R_t}}{\partial Y_t} Y^H + \overline{MPC}(\widehat{R_t}(Y_t)) - \widehat{R^*}^{-1} \chi(\widehat{R^*}) Y^H \frac{\partial \widehat{R_t}}{\partial Y_t} \right] \right]^{-1}$$

We note $a_1 > 0$ by the assumption that $\frac{\partial Spend_t}{\partial Y_t} < 1$.

$$\begin{bmatrix} Y_t \\ \widehat{Y_{t+1}} \end{bmatrix} = \begin{bmatrix} (\nu + \chi\lambda)a_1 & \chi(1-\lambda)a_1 \\ \lambda & (1-\lambda) \end{bmatrix} \begin{bmatrix} Y_{t-1} \\ \widehat{Y}_t \end{bmatrix} \tag{A.17}$$

The characteristic equation is then given by

$$L(\Theta) = \Theta^2 - \Theta(\nu+\chi\lambda)a_1 - \Theta(1-\lambda) + (\nu+\chi\lambda)(1-\lambda)a_1 - \chi(1-\lambda)\lambda a_1 \tag{A.18}$$

At $L(0)$, characteristic equation is positive

$$(\nu + \chi\lambda)(1-\lambda)a_1 - \chi(1-\lambda)\lambda a_1 > 0 \tag{A.19}$$

At $L(1)$, the characteristic equation is

$$\begin{aligned}1-(\nu+\chi\lambda)a_1-(1-\lambda)+(\nu+\chi\lambda)(1-\lambda)a_1-\chi(1-\lambda)\lambda a_1\\ \lambda-\lambda(\nu+\chi\lambda)a_1-\chi(1-\lambda)\lambda a_1\\ \lambda(1-(\nu+\chi\lambda)a_1-\chi(1-\lambda)a_1)\\ \lambda(1-\chi a_1-\nu a_1)\end{aligned}$$

We can bound this as follows. The PE Keynesian multiplier is given by

$$\alpha(R^*)=\frac{1}{1-\overline{MPC}(\widehat{R^*})}>a_1 \tag{A.20}$$

We thus have

$$\lambda(1-\chi a_1-\nu a_1)>\lambda(1-(\chi+\nu)\alpha(R^*))$$

However, from equation A.10, $(\chi+\nu)\alpha(R^*)=1$. We thus have

$$\lambda(1-\chi a_1-\nu a_1)>\lambda(1-(\chi+\nu)\alpha(R^*))=0$$

At $L(1)$, the characteristic equation is positive. We thus have both eigenvalues less than 1 in absolute value.

□

Proof of proposition 3.

Proof. The model with government spending is given by

$$\begin{aligned}Y_t=\alpha(\widehat{R_t})\Big(\nu Y_{t-1}+\chi(\widehat{R_t})\widehat{Y_{t+1}}+\\ \Theta(\tau_t)G_t-\Upsilon(\tau_t)B^G_t+\Upsilon(\tau_t)\frac{B^G_{t+1}}{\widehat{R_t}}\Big)\end{aligned} \tag{A.21}$$

Here $\Upsilon(\tau_t)=\tau^y_t+\frac{1}{1+\beta}\tau^m_t$, $\Theta(\tau_t)=1-\tau^y_t-\frac{1}{1+\beta}\tau^m_t$.

We consider the simplest case, with $B^G_s=0\;\forall s$, and $\Theta(\tau_t)>0$, i.e. $\tau^m_t>0$.

The proof then follows from the argument from the proof of 1. We will have $\frac{\partial Y_t}{\partial G_t}>0$ when the GE Keynesian cross has a slope less than one, which holds when equation A.14 is negative.

□

Proof of proposition 4.

Proof. From the Fisher equation,

$$\widehat{R_t}(Y_t,\epsilon^i_t)=\frac{1}{\pi_t}\left[max(1,(1+i^*)(\frac{\pi_t(Y_t)}{\pi^*})^{\phi_\pi})+\epsilon^i_t\right] \tag{A.22}$$

We thus have

$$\frac{\partial\widehat{R_t}}{\partial\epsilon^i_t}>0 \tag{A.23}$$

The logic of the proof then follows the logic of the proof of proposition 1. We will have $\frac{\partial Y_t}{\partial \epsilon_t^i} > 0$ when the GE Keynesian cross has a slope less than one, which holds when equation A.14 is negative.

□

Proof of proposition 5.

Proof. From the IS-Exp equation, the dynamical system can be written

$$Y_t = \nu Y_{t-1} + \overline{MPC}(\widehat{R_t})Y_t + \chi(\widehat{R_t})\widehat{Y_{t+1}}$$
$$\widehat{Y_{t+1}} = \lambda Y_{t-1} + (1-\lambda)\widehat{Y}_t + \epsilon_t^e$$

We linearize the dynamical system around the high steady state

$$\frac{\partial}{\partial Y_t} = \frac{\partial \overline{MPC}}{\partial \widehat{R_t}}\frac{\partial \widehat{R_t}}{\partial Y_t}Y_t + \overline{MPC}(\widehat{R_t}(Y_t)) - \widehat{R_t}^{-1}\chi(\widehat{R_t})\widehat{Y_{t+1}}\frac{\partial \widehat{R_t}}{\partial Y_t}$$

With a peg, $R_t = I^*/\pi_t$, and thus $\frac{\partial \widehat{R_t}}{\partial Y_t} = -I^*\pi_t^{-2}\frac{\partial \pi_t}{\partial Y_t} = -I^*\pi_t^{-2}\kappa < 0$.

$$\tilde{Y}_t = \left[\frac{\partial \overline{MPC}}{\partial \widehat{R_t}}\frac{\partial \widehat{R_t}}{\partial Y_t}Y^H + \overline{MPC}(\widehat{R^*}) - \widehat{R^*}^{-1}\chi(\widehat{R^*})Y^H\frac{\partial \widehat{R_t}}{\partial Y_t}\right]\tilde{Y}_t$$
$$+[\nu + \chi(R^*)\lambda]\widetilde{Y_{t-1}} + \chi(R^*)(1-\lambda)\tilde{\widehat{Y}}_t$$

We define the GE Keynesian multiplier a_1 as

$$a_1 = \left[1 - \left[\frac{\partial \overline{MPC}}{\partial \widehat{R_t}}\frac{\partial \widehat{R_t}}{\partial Y_t}Y^H + \overline{MPC}(\widehat{R^*}) - \widehat{R^*}^{-1}\chi(R^*)Y^H\frac{\partial \widehat{R_t}}{\partial Y_t}\right]\right]^{-1}.$$

We note $a_1 > 0$ by the assumption that the slope of the GE expenditure function is less than 1: $\frac{\partial Spend_t}{\partial Y_t} < 1$.

The dynamical system is given by

$$\begin{bmatrix} Y_t \\ \widehat{Y_{t+1}} \end{bmatrix} = \begin{bmatrix} (\nu+\chi\lambda)a_1 & \chi(1-\lambda)a_1 \\ \lambda & (1-\lambda) \end{bmatrix}\begin{bmatrix} Y_{t-1} \\ \widehat{Y}_t \end{bmatrix} \tag{A.24}$$

The characteristic equation is then given by

$$L(\Theta) = \Theta^2 - \Theta(\nu+\chi\lambda)a_1 - \Theta(1-\lambda) + (\nu+\chi\lambda)(1-\lambda)a_1 - \chi(1-\lambda)\lambda a_1 \tag{A.25}$$

At $L = 0$, characteristic equation is positive

$$(\nu+\chi\lambda)(1-\lambda)a_1 - \chi(1-\lambda)\lambda a_1 > 0 \tag{A.26}$$

At $L = 1$, the characteristic equation is

$$\begin{aligned} 1 - (\nu + \chi\lambda)a_1 - (1-\lambda) + (\nu + \chi\lambda)(1-\lambda)a_1 - \chi(1-\lambda)\lambda a_1 \\ \lambda - \lambda(\nu + \chi\lambda)a_1 - \chi(1-\lambda)\lambda a_1 \\ \lambda(1 - (\nu + \chi\lambda)a_1 - \chi(1-\lambda)a_1) \\ \lambda(1 - \chi a_1 - \nu a_1) \end{aligned}$$

We can bound this as follows. The PE Keynesian multiplier is given by

$$\alpha(R^*) = \frac{1}{1 - \overline{MPC}(\widehat{R_t}(Y_t))} < a_1 \tag{A.27}$$

We thus have

$$\lambda(1 - \chi a_1 - \nu a_1) < \lambda(1 - (\chi - \nu)\widehat{a_1})$$

However, from equation A.10, $(\chi + \nu)\alpha(R^*) = 1$. We thus have

$$\lambda(1 - \chi a_1 - \nu a_1) < \lambda(1 - (\chi - \nu)\widehat{a_1}) = 0$$

At $L = 1$, the characteristic equation is negative. We thus have one Eigenvalue less than 1 in absolute value, and one greater than one.

□

Proof of Propositions 6.

Proof. Under rational expectations, the dynamical system is given by the IS-Exp curve,

$$Y_{t+1} = \Big[(1 - \overline{MPC}(\widehat{R_t}))Y_t - \nu Y_{t-1}\Big]\chi(\widehat{R_t})^{-1} \tag{A.28}$$

We perform a change in variables, defining $Z_{t+1} = Y_t$. The full system is then

$$\begin{aligned} Y_{t+1} = \Big[(1 - \overline{MPC}(\widehat{R_t}))Y_t - \nu Z_t\Big]\chi(\widehat{R_t})^{-1} \\ Z_{t+1} = Y_t. \end{aligned}$$

This is a non-linear system. To analyze the stability, we linearize the model around the high steady state. The coefficient for Y_t is given by

$$\begin{aligned} \partial Y_{t+1}/\partial Y_t = (1 - \overline{MPC}(\widehat{R_t}))\chi(\widehat{R_t})^{-1} \\ + Y_t\beta\frac{\gamma}{1-\gamma}\kappa\widehat{R_t}\frac{1}{\pi_t}(\phi_\pi - 1) \\ - Z_t\frac{\beta D}{1-\gamma}\kappa\widehat{R_t}\frac{1}{\pi_t}(\phi_\pi - 1). \end{aligned}$$

The term for Z_t is

$$\partial Y_{t+1}/\partial Z_t = -\nu\chi(\widehat{R_t})^{-1}.$$

At the high steady state, $Y_t = 1$, $\pi = \pi^*$, $Z_t = 1$, and thus

$$\begin{aligned}
\partial Y_{t+1}/\partial Y_t &= (1 - \overline{MPC}(\widehat{R^*}))\chi(\widehat{R^*})^{-1} \\
+\beta\frac{\gamma}{1-\gamma}R^*\frac{1}{\pi^*}(\phi_\pi - 1)\kappa - \frac{\beta D}{1-\gamma}R^*\frac{1}{\pi^*}(\phi_\pi - 1)\kappa \\
&= (1 - \overline{MPC}(\widehat{R^*}))\chi(\widehat{R^*})^{-1} + \\
&\quad \frac{1}{1-\gamma}\beta(\gamma - D)R^*\frac{1}{\pi^*}(\phi_\pi - 1)\kappa \\
&\equiv (1 - \overline{MPC}(\widehat{R^*}))\chi(\widehat{R^*})^{-1} + \Gamma(\phi_\pi)
\end{aligned}$$

The linearized system in deviations from steady state is given by

$$\begin{bmatrix} Y_{t+1} \\ Z_{t+1} \end{bmatrix} = \begin{bmatrix} (1 - \overline{MPC}(\widehat{R^*}))\chi(\widehat{R^*})^{-1} + \Gamma(\phi_\pi) & -\nu\chi(\widehat{R^*})^{-1} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} Y_t \\ Z_t \end{bmatrix} \tag{A.29}$$

The Eigenvalues are given by the zeros of the characteristic equation

$$L(\Theta) = \Theta^2 - \Theta\left[(1 - \overline{MPC}(\widehat{R^*}))\chi(\widehat{R^*})^{-1} + \Gamma(\phi_\pi)\right] + \nu\chi(\widehat{R^*})^{-1} \tag{A.30}$$

At $L(0)$, the left hand side of equation A.30 is positive. In order for there to be one unstable Eigenvalue, it must be the case that $L(1)$ is negative.

At $L(1)$, the characteristic equation A.30 equals

$$\begin{aligned}
L(1) &= 1 - (1 - \overline{MPC}(\widehat{R^*}))\chi(\widehat{R^*})^{-1} + \nu\chi(\widehat{R^*})^{-1} - \Gamma(\phi_\pi) \\
&= 1 - \chi(\widehat{R^*})^{-1}\left((1 - \overline{MPC}(\widehat{R^*})) - \nu\right) - \Gamma(\phi_\pi) \\
&= 1 - \chi(\widehat{R^*})^{-1}\chi(\widehat{R^*}) - \Gamma(\phi_\pi) = -\Gamma(\phi_\pi)
\end{aligned}$$

The last equality follows from equation A.10. If $\phi_\pi > 1$, $\Gamma(\phi_\pi) > 0$, and thus $L(1) < 0$, and there will be exactly one stable and one unstable Eigenvalue. If $\phi_\pi < 1$, there will be two stable Eigenvalues. □

Proof of Propositions 7.

Proof. The steps are the same as in proposition 6, except now the derivatives are taken around the low steady state. At the low steady state, the real interest rate is unchanged, but the nominal interest rate is 1. From the Fisher equation, we then have $\pi_t = R^{*-1}$. Using the aggregate supply curve, $\pi^L = \left(\pi^* - \kappa(1 - Y^L)\right) = (R^*)^{-1}$. Thus $Y^L = \frac{1}{\kappa}\left((R^*)^{-1} - \pi^L\right) + 1$.

We linear the IS-Exp curve. The coefficient for Y_t is given by

$$\begin{aligned}
\partial Y_{t+1}/\partial Y_t &= (1 - \overline{MPC}(\widehat{R_t}))\chi(\widehat{R_t})^{-1} \\
&\quad + Y_t\beta\frac{\gamma}{1-\gamma}\frac{\partial\widehat{R_t}}{\partial Y_t} \\
&\quad - Z_t\frac{\beta D}{1-\gamma}\frac{\partial\widehat{R_t}}{\partial Y_t}.
\end{aligned}$$

The terms for Z_t are

$$\partial Y_{t+1}/\partial Z_t = -\nu\chi(\widehat{R_t})^{-1}$$

At the low steady state, $\frac{\partial \widehat{R_t}}{\partial Y_t} = -\kappa R^*/\pi^L = -\kappa R^{*2}$.

$$\begin{aligned}\partial Y_{t+1}/\partial Y_t = (1-\overline{MPC}(\widehat{R^*}))\chi(\widehat{R^*})^{-1}\\ -Y^L\beta\frac{\gamma}{1-\gamma}\kappa R^{*2}\\ +Y^L\frac{\beta D}{1-\gamma}\kappa R^{*2} =\\ (1-\overline{MPC}(\widehat{R^*}))\chi(\widehat{R^*})^{-1} + (D-\gamma)Y^L\beta\frac{1}{1-\gamma}\kappa R^{*2}\\ \equiv (1-\overline{MPC}(\widehat{R^*}))\chi(\widehat{R^*})^{-1} + \Gamma\end{aligned}$$

The terms for Z_t / evaluated at the steady state is

$$\partial Y_{t+1}/\partial Z_t = -\nu\chi(\widehat{R^*})^{-1}$$

The linearized system in deviations from steady state is given by

$$\begin{bmatrix} Y_{t+1} \\ Z_{t+1}\end{bmatrix} = \begin{bmatrix}(1-\overline{MPC}(\widehat{R^*}))\chi(\widehat{R^*})^{-1}+\Gamma & -\nu\chi(\widehat{R^*})^{-1} \\ 1 & 0\end{bmatrix}\begin{bmatrix} Y_t \\ Z_t\end{bmatrix} \tag{A.31}$$

The Eigenvalues are given by the zeros of the characteristic equation

$$L(\Theta) = \Theta^2 - \Theta\left[(1-\overline{MPC}(\widehat{R^*}))\chi(\widehat{R^*})^{-1}+\Gamma\right] + \nu\chi(\widehat{R^*})^{-1} \tag{A.32}$$

At $L(0)$, the left hand side of equation A.32 is positive. In order for there to be two stable Eigenvalues, it must be the case that $L(1)$ is positive. We have

$$\begin{aligned}L(1) &= 1-(1-\overline{MPC}(\widehat{R^*}))\chi(\widehat{R^*})^{-1} + \nu\chi(\widehat{R^*})^{-1} - \Gamma\\ &= 1-\chi(\widehat{R^*})^{-1}\left((1-\overline{MPC}(\widehat{R^*}))-\nu\right) - \Gamma\\ &= 1-\chi(\widehat{R^*})^{-1}\chi(\widehat{R^*}) - \Gamma = -\Gamma\end{aligned}$$

As long as $\gamma > D$, Γ will be negative and thus L(1) will be positive, and thus both eigenvalues will be less than 1 in absolute value.

□

Proof of Propositions 8.

Proof. The steps are similar to those in proposition 6.

We linearize the IS-Exp curve. The coefficient for Y_t is given by

$$\begin{aligned}\partial Y_{t+1}/\partial Y_t = (1-\overline{MPC}(\widehat{R_t}))\chi(\widehat{R_t})^{-1}\\ +Y_t\beta\frac{\gamma}{1-\gamma}\frac{\partial\widehat{R_t}}{\partial Y_t}\\ -Z_t\frac{\beta D}{1-\gamma}\frac{\partial\widehat{R_t}}{\partial Y_t}.\end{aligned}$$

The terms for Z_t are

$$\partial Y_{t+1}/\partial Z_t = -\nu\chi(\widehat{R_t})^{-1}$$

With a peg, $R_t = I^*/\pi_t$, and thus $\frac{\partial \widehat{R_t}}{\partial Y_t} = -I^*\pi_t^{-2}\frac{\partial \pi_t}{\partial Y_t} = -I^*\pi_t^{-2}\kappa = -R^*\pi_t^{-1}\kappa < 0$.

$$\begin{aligned}\partial Y_{t+1}/\partial Y_t = (1-\overline{MPC}(R^*))\chi(R^*)^{-1}\\ -\beta\frac{\gamma}{1-\gamma}R^*\pi^{*-1}\kappa\\ +\frac{\beta D}{1-\gamma}R^*\pi^{*-1}\kappa =\\ (1-\overline{MPC}(R^*))\chi(R^*)^{-1} + (D-\gamma)1\beta\frac{1}{1-\gamma}R^*\pi^{*-1}\kappa\\ \equiv (1-\overline{MPC}(R^*))\chi(R^*)^{-1} + \Gamma\end{aligned}$$

The terms for Z_t / evaluated at the steady state is

$$\partial Y_{t+1}/\partial Z_t = -\nu\chi(R^*)^{-1}$$

The linearized system in deviations from steady state is given by

$$\begin{bmatrix} Y_{t+1} \\ Z_{t+1}\end{bmatrix} = \begin{bmatrix}(1-\overline{MPC}(\widehat{R^*}))\chi(\widehat{R^*})^{-1}+\Gamma & -\nu\chi(\widehat{R^*})^{-1} \\ 1 & 0\end{bmatrix}\begin{bmatrix} Y_t \\ Z_t\end{bmatrix} \tag{A.33}$$

The Eigenvalues are given by the zeros of the characteristic equation

$$L(\Theta) = \Theta^2 - \Theta\left[(1-\overline{MPC}(\widehat{R^*}))\chi(\widehat{R^*})^{-1}+\Gamma\right] + \nu\chi(\widehat{R^*})^{-1} \tag{A.34}$$

At $L(0)$, the left hand side of equation A.34 is positive. In order for there to be two stable Eigenvalues, it must be the case that $L(1)$ is positive. We have

$$\begin{aligned}L(1) &= 1-(1-\overline{MPC}(\widehat{R^*}))\chi(\widehat{R^*})^{-1} + \nu\chi(\widehat{R^*})^{-1} - \Gamma\\ &= 1-\chi(\widehat{R^*})^{-1}\left((1-\overline{MPC}(\widehat{R^*}))-\nu\right)-\Gamma\\ &= 1-\chi(\widehat{R^*})^{-1}\chi(\widehat{R^*}) - \Gamma = -\Gamma\end{aligned}$$

As long as $\gamma > D$, Γ is negative, and thus $L(1)$ is positive both eigenvalues will be less than 1 in absolute value.

□

Proof of proposition 10.

The dynamical system under government spending can be written

$$Y_{t+1} = \left[(1-\overline{MPC}(\widehat{R_t}))Y_t - \nu Z_t - G_t\Theta(\tau_t) - B_t^G\Theta(\tau_t) - \frac{B_{t+1}^G}{\widehat{R_t}}\right]\chi(\widehat{R_t})^{-1} \tag{A.35}$$

$$Z_{t+1} = Y_t. \tag{A.36}$$

Let Y_s^0 be the level of output along the saddle path for the system with zero government spending, and Y_s^{temp} the level of output for the path with a one time increase in government spending in period t. From equation A.35, from period $t+2$ onwards the dynamical system of both paths are the same, since there is no govenrment spending or debt after period t. Only in period $t+1$ is there a difference in the dynamical systems, from the temporary spending. The question of the difference between the two then boils down to: is $Y_t^{temp} > Y_t^0$?

Consider what would happen if $Y_t^{temp} = Y_t^0$. From equation $A.35$, we would then have $Y_{t+1}^{temp} < Y_{t+1}^0$. However, this would mean that the path with government spending would be below the saddle path to the high steady state. The transition path where $Y_t^{temp} = Y_t^0$ is shown in figure 7a, teal line. It is thus clear that we must have $Y_t^{temp} > Y_t^0$ on the saddle path.

We can actually say even more about the path of government spending: $Y_s^{temp} > Y_s^0 \ \forall s \geq t+1$. To do this, we will first prove a lemma that shows if we hold Z_t fixed, $\frac{\partial Y_{t+n}}{\partial Y_t} > 0$. At period $t+2$ we have $Y_{t+2}^{temp} > Y_{t+2}^0$ with the same Z_t, and thus

Lemma: $\frac{\partial Y_{t+n}}{\partial Y_t}$, holding Z_t fixed.

From the proof of proposition 6, the dynamical system is given by

$$\begin{bmatrix} Y_{t+1} \\ Z_{t+1} \end{bmatrix} = \begin{bmatrix} (1-\overline{MPC}(\widehat{R^*}))\chi(\widehat{R^*})^{-1} + \Gamma(\phi_\pi) & -\nu\chi(\widehat{R^*})^{-1} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} Y_t \\ Z_t \end{bmatrix} \tag{A.37}$$

We define the following variables:

$$\begin{aligned} a &= (1-\overline{MPC}(\widehat{R^*}))\chi(\widehat{R^*})^{-1} + \Gamma(\phi_\pi) \\ b &= -\nu\chi(\widehat{R^*})^{-1} \end{aligned}$$

The dynamical system can thus be written

$$\begin{bmatrix} Y_{t+1} \\ Z_{t+1} \end{bmatrix} = \begin{bmatrix} a & b \\ 1 & 0 \end{bmatrix} \begin{bmatrix} Y_t \\ Z_t \end{bmatrix} \tag{A.38}$$

Performing a Jordan decomposition, we have

$$\begin{bmatrix} Y_{t+1} \\ Z_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(a-\zeta) & \frac{1}{2}(a+\zeta) \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2}(a-\zeta) & 0 \\ 0 & \frac{1}{2}(a+\zeta) \end{bmatrix} \begin{bmatrix} -\frac{1}{\zeta} & \frac{1}{2}(1+\frac{a}{\zeta}) \\ \frac{1}{\zeta} & \frac{1}{2}(1-\frac{a}{\zeta}) \end{bmatrix} \begin{bmatrix} Y_t \\ Z_t \end{bmatrix}$$

Where $\zeta = \sqrt{a^2+4b}$. Forwarding the system, we have

$$\begin{aligned} \begin{bmatrix} Y_{t+n} \\ Z_{t+n} \end{bmatrix} &= \begin{bmatrix} \frac{1}{2}(a-\zeta) & \frac{1}{2}(a+\zeta) \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2}(a-\zeta) & 0 \\ 0 & \frac{1}{2}(a+\zeta) \end{bmatrix}^n \begin{bmatrix} -\frac{1}{\zeta} & \frac{1}{2}(1+\frac{a}{\zeta}) \\ \frac{1}{\zeta} & \frac{1}{2}(1-\frac{a}{\zeta}) \end{bmatrix} \begin{bmatrix} Y_t \\ Z_t \end{bmatrix} \\ &= \begin{bmatrix} \left(\frac{1}{2}\right)^{n+1}(a-\zeta)^{n+1} & \left(\frac{1}{2}\right)^{n+1}(a+\zeta)^{n+1} \\ \left(\frac{1}{2}\right)^{n}(a-\zeta)^{n} & \left(\frac{1}{2}\right)^{n}(a+\zeta)^{n} \end{bmatrix} \begin{bmatrix} -\frac{1}{\zeta} & \frac{1}{2}(1+\frac{a}{\zeta}) \\ \frac{1}{\zeta} & \frac{1}{2}(1-\frac{a}{\zeta}) \end{bmatrix} \begin{bmatrix} Y_t \\ Z_t \end{bmatrix} \end{aligned}$$

We thus have

$$\frac{\partial Y_{t+n}}{\partial Y_t} = \left[-\frac{1}{\zeta} \cdot \left(\frac{1}{2}\right)^{n+1} (a-\zeta)^{n+1} + \frac{1}{\zeta} \left(\frac{1}{2}\right)^{n+1} (a+\zeta)^{n+1} \right]$$

We know $\zeta > 0$ because the Eigenvalues of the system are real, not imaginary. The above expression is thus positive.

C List of Symbols

- Quantities
  - B_t^m, B_t^o: face value of real bonds for middle aged and old purchased at time $t-1$ at a price $1/\widehat{R_{t-1}}$
  - B_{t+1}^G: face value of real government bonds purchased at time t at price $1/\widehat{R_t}$
  - C_t^y, C_t^m, C_t^o: consumption of young, middle aged, old agents
  - $\widehat{C_{t+1}^o}$: subjective expectations of consumption in old age
  - G_t: government spending
  - L_t: labor demand/supply
  - Π_t: profits of monopolistically competitive firms, distributed to middled aged Π_t^m and Π_t^o in proportion to γ
  - U_t: subjective utility of agents
  - $Spend_t(Y_t)$: aggregate expenditure function, which is a function of income
  - T_t: total taxes
  - τ^y, τ^m, τ^o: fraction of taxes paid by the different generations.
  - Y_t: income and output
  - Y^*: full employment, equal to 1. Also referred to as Y^H.
  - $\widehat{Y_{t+1}}$: subjective expectations of output
  - $y_t^f(i)$: differentiated final goods in the CES aggregate, indexed by i.
  - Y^L, Y^H: output in the high and low steady states.
  - Y^0: output in the "zero" steady state
  - Y_t^U: Unintended steady state in the Mertens & Ravn type model.
  - $\overline{Y^U}$: in modified Mertens & Ravn model, agents expectation of output in unintended steady state.
- Prices

- I_t: gross nominal interest rate
- $p_t(i)$: price of differentiated final good, indexed by I
- P_t: nominal price index of final good aggregate. $P_t = \left(\int_0^1 p_t\,(i)^{1-\Lambda_t}\,di\right)^{\frac{1}{1-\Lambda_t}}$.
- $\widehat{\pi_{t+1}}$: subjective expectation of inflation
- π^*: central bank's inflation target
- π^H: inflation in the high steady state, equal to π^*
- π^L: inflation in the low steady state, equal to $1/R^*$.
- $\widehat{R_t}$: expected real interest rate, equal to $I_t\widehat{\pi_{t+1}}$
- w_t: real wage

• Parameters

- a_1, a_2: parameters in the original specification of the Phillips curve: $\pi_t - \pi^* = a_1(Y_t - Y^*) + a_2(\widehat{\pi_{t+1}} - \pi^*)$.
- $\alpha(\widehat{R_t})$: partial equilibrium Keynesian multiplier, equal to $\frac{1}{1-\overline{MPC}(\widehat{R_t})}$. Exact formula $\alpha(\widehat{R_t}) = \frac{1}{\left[1-mpc^y\frac{D}{\widehat{R_t}}-mpc^m\gamma-mpc^o(1-\gamma)\right]}$.
- D_t: debt limit for young agents.
- γ: fraction of labor supplied by middle age
- $(1+i^*)$: full employment nominal interest rate given inflation target, equal to $R^*\pi^*$.
- κ: slope of Phillips curve, $\kappa = a_1/(1-a_2)$.
- λ: adaptive expectations updating parameter
- Λ: elasticity of substitution of CES aggregate
- $\overline{MPC}(\widehat{R_t}) = mpc^y\frac{D}{\widehat{R_t}} + mpc^m\gamma + mpc^o(1-\gamma)$. $\overline{MPC}(\widehat{R_t})$ is a weighted average of the individual generations' marginal propensities to consume. The weights are relative shares of income of the different generations.
- mpc^y, mpc^m, mpc^o: The marginal propensities to consume of the young, middle aged, and old are 1, $\frac{1}{1+\beta}$, and 1, respectively.
- μ: markup of monopolistically competitive firms, $\mu = (\Lambda/(\Lambda-1)$.
- $\nu = \frac{\beta D}{1+\beta}$. This is the aggregate marginal propensity to consume out of income last period.
- ϕ_π: Taylor rule coefficient.
- $\Theta(\tau_t) = 1-\tau_t^y-\frac{1}{1+\beta}\tau_t^m$. Parameter in IS-Exp curve with government spending, determines the size of government multiplier.

- $\Upsilon(\tau_t) = \tau_t^y + \frac{1}{1+\beta}\tau_t^m$. Parameter in IS-Exp curve with government spending, affects how government borrowing influences the multiplier.
- $\chi(\widehat{R_t}) = mpc^m(1-\gamma)/\widehat{R_t}$: the marginal propensity to consume out of expected future income.
- ξ: extrapolative expectations parameter.
- z: In Mertens & Ravn type model, the probability output will remain low.

• Shocks

- ϵ_t^e: shock to expectations
- ϵ_t^i: monetary policy shock to nominal interest rates